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# Merging Platforms:

## a study of horizontal mergers in two-sided markets

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**Abstract**

This thesis provides a theoretical analysis of how a horizontal merger affects consumer welfare in digital platform markets. I consider a model of two competing platforms and examine how consumer surplus is affected by a merger under two possible scenarios: (i) a merger into a joint ownership of the two platforms, and (ii) the case where the merged entity shuts down one platform. I find that the merged entity's decision to keep or not to keep both platforms, is determined by the scope of the related fixed costs. The analysis shows that consumer surplus cannot increase as a result of a merger into a joint ownership of the platforms. In case the merged entity shuts down one platform the effects on consumer welfare are ambiguous.

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# 1 Introduction

As more and more economic activities take place virtually, two-sided markets (also referred to as 'platform markets') have become increasingly prevalent. Examples are e-commerce marketplaces, digital media markets (such as newspapers, magazines and streaming services), online advertising platforms and online intermediation of real estate, travels and jobs. At the same time, antitrust authorities have become increasingly concerned with the high concentration in these types of markets, not least in the case of horizontal mergers. The aim of this thesis is to analyze how a horizontal merger affects consumer surplus in digital platform markets.

There are two core attributes that distinguish two-sided markets from standard one-sided markets. First, there is a two-sided aspect since a platform serves two different groups of consumers. Second, there are indirect network effects: greater involvement by consumers in one group increases the value of the platform to consumers in the other group. A platform creates value by internalizing these network effects and thereby enabling the interaction between the groups.

In the presence of network effects, consumers tend to value larger market players, sometimes to the extent that all consumers end up joining the same platform (Caillaud & Jullien 2003). Two-sided markets therefore tend to be highly concentrated. In contrast to one-sided markets, less competition may not imply less efficient market allocations.

While it is widely recognized that the one-sided logic is therefore inappropriate for the analysis of mergers in two-sided markets, the theoretical literature on the effects of horizontal mergers in two-sided markets is quite scarce. The focus in both theoretical and empirical merger analyzes has often been on the newspaper industry. The aim of this thesis is to extend this line of research by considering a merger between digital platforms more generally. My analysis departs from previous theoretical attempts to address mergers in two-sided markets in two ways. First, I do not assume a specific relationship between the consumer groups' respective preferences over platform types or intensity of indirect network effects.[1] Second, I address the

[1]In Leonello (2011) indirect network effects are assumed to be present on only one side of the

incentives for the merged entity to keep both platforms under a joint ownership versus to shut down one platform and operate the remaining one. In contrast to Tan & Zhou (2019) that conclude that merging platforms have no incentives to shut down one platform, I find that the profits can be higher for a merged entity that shuts down one platform if the fixed costs associated with the maintenance of a platform are high.

Fixed costs are indeed of importance in digital two-sided markets. Most online platform markets are characterized by high fixed costs and near zero marginal costs (Duch-Brown 2017). It is therefore relevant to consider the role of fixed costs in the merged entity's decision to keep, or not to keep, both platforms. For the same reason it is plausible to assume that platforms' incentives to merge are primarily motivated by savings in fixed costs.

As already mentioned, indirect network effects may lead to a monopoly situation in platform markets. More platforms can co-exist in a two-sided market when platforms offer differentiated services or when consumers have the possibility to patronize more than one platform (Evans & Schmalensee 2017). In line with several previous studies, I consider a merger in a market with differentiated platforms and assume that consumers of both groups join only one platform.[2] Specifically, I consider a merger between two digital platforms that are horizontally differentiated in terms of the services they provide and where consumers have heterogeneous preferences over platform types. In markets where consumers of one group only value a platform by its ability to enable the interaction with consumers of the other group (as is the case in e.g. e-commerce marketplaces) the horizontal differentiation can be reflected in differences in layout or other features affecting the platform experience. In markets where platforms provide content in addition to the ability to interact on the platform (as is the case in e.g. online advertising markets, the digital newspapers industry or online markets for travel intermediation) the platforms may differ in what type of content they provide (consider for example two travel intermediation platforms that offer information on different types of travel destinations and

market. Leonello (2011) and Cosnita-Langlais et al. (2016) assume that the groups have perfectly symmetric preferences over platform types

[2]In the literature, this is referred to as 'single-homing' whereas the case where consumers patronize more than one platform is referred to as 'multi-homing'.

thereby attract different types of consumers).

Initially, the mere purpose of this thesis was to perform a merger analysis in the described setting. To derive the pre-merger equilibrium I followed a model of two competing platforms located at the endpoints of the Hotelling line presented in a seminal article by Armstrong (2006). This model is well-established and has been adapted in previous studies on mergers in two-sided markets. I found that the model may be inappropriate for a merger analysis unless it is slightly modified.

For a merger analysis to be relevant the competing platforms must share a market segment where they compete for consumers. In the opposite case, the platforms enjoy positions as local monopolists and the lack of strategic interaction between the platforms implies that their profit maximizing strategies will be unaffected by a merger. In a duopoly setting with platforms located at the extreme points of the Hotelling line this implies that the assumption of a covered market must be satisfied. I find that this assumption may be incompatible with the necessary condition for a unique market sharing equilibrium in the model proposed by Armstrong (2006) which had two major implications for the thesis. Firstly, I conclude that the model is not useful for the analysis of markets where consumers of one group only value a platform by its ability to enable the interaction with consumers of the other group. In particular I find that consumers on at least one side of the market must obtain some benefit from joining a platform in addition to the benefit of interacting on the platform in order to ensure compatibility between the conditions for a covered market and the condition for a unique market sharing equilibrium. Such stand-alone benefits can be motivated if the platforms offer some content in addition to the provision of interaction possibilities. Second, the finding that the absence of stand-alone benefits implies a contradiction between these conditions is intrinsically interesting and the question arises whether horizontal differentiation is sufficient to explain competition in two-sided markets absent such benefits.

This thesis is therefore divided into two parts. In the first part I present Armstrong's (2006) model of competing platforms and show that stand-alone benefits are necessary for an adequate derivation of the pre-merger equilibrium. In the second part I address a horizontal merger in this setting and derive the post-merger equilibrium under two possible scenarios: a merger into a joint ownership of the two platforms and the case where the merged entity shuts down one platform. While my primary aim is to analyze the effects of a merger on consumer welfare, I will also give some attention to the results derived in the first part of the thesis. In particular, I discuss what we may conclude from the observation that the condition for a unique duopoly equilibrium and the condition for overlapping market areas are contradictory in case users only value a platform by its ability to enable the interaction with users of another group.

As for the merger analysis, the focus is exclusively on possible impacts on prices and thereby the consumer surplus. This is motivated by the fact that antitrust authorities tend to use the consumer surplus, and not the total surplus, as the relevant welfare measure in merger assessments. There are obviously a number of other reasons for why antitrust authorities may be concerned with mergers in concentrated markets. The EU Horizontal Merger Guidelines mention reduced choice, deterioration in quality as well as reduced incentives for innovation. Such effects are not considered in this thesis.

The remainder of this thesis is organized as follows. In section 2 I overview the existing literature on two-sided markets generally, and on horizontal mergers particularly. In sections 3 and 4, which constitute the first part of this thesis, I present a model of competing platforms proposed by Armstrong (2006) and show that standalone benefits for at least one side of the market are necessary for the equilibrium to be compatible with the assumption of a covered market. Sections 5 and 6 discuss the choices and the decisions of the merged entity. In section 7 I analyze the effects of a horizontal merger on the consumer surplus. Finally, section 8 discusses the main findings and conclusions as well as the limitations of the analysis conducted in this thesis.

## 2 Literature Review

The observation that the one-sided market logic is inappropriate to explain the economic characteristics of two-sided industries was first made by Rochet & Tirole (2003). By combining elements from the literature on network externalities, that mainly focuses on user adoption and network size, and the multiproduct literature that stresses product cross-elasticities, the authors laid the foundations for the subsequent literature on two-sided markets. The authors emphasize the price structure of market intermediaries (or platforms).[3] In particular, they show that both privately and socially optimal pricing can entail below marginal cost pricing on one side, and higher markups on the other side because the price structure is set as to bring both sides of the markets "on board". Other seminal work include Caillaud & Jullien (2003), Rochet & Tirole (2006) and Armstrong (2006).[4] A substantial part of the recent literature on two-sided markets builds on these early contributions.

Caillaud & Jullien (2003) analyze platform markets in an environment where intermediaries take the roles as matchmakers. They consider the case where agents have a higher valuation for a matchmaker with a large pool of agents of the other group since that increases the probability of a match. It is shown that when homogeneous intermediaries compete in fixed fees and usage fees, the only equilibrium is that one of the intermediaries attracts all consumers and that this outcome is efficient.

In other environments agents value a matchmaker by the composition of its pool of agents rather than its size. Damiano & Li (2008) find that matchmaking platforms can control the composition of their pools of agents by the sorting role of prices. The vertical platform differentiation that arises enables two platforms to coexist in equilibrium. Indeed, vertical and horizontal differentiation are primary reasons for the occurrence of competition in two-sided markets (Evans & Schmalensee 2017).

In other settings two different groups of agents may interact through an intermediary (or a platform) and the utility of one group increases in the size of the other group. An intermediary creates value by bringing the two sides together. This applies for many traditional markets such as the credit card market and flea markets as well as digitized markets such as e-commerce marketplaces, digital media markets (newspapers, magazines and streaming services), online advertising platforms

[3]In an overview article over the early literature on two-sided markets, Rysman (2009) asserts that it is indeed the focus on market intermediaries and in particular their pricing strategies that mainly distinguishes the literature on two-sided markets from the literature on network effects.

[4]Another influential paper that develops the analysis in Rochet & Tirole (2003, 2006) is Weyl (2010) who analyzes monopoly pricing in platform markets. He uses insulated tariffs which allows for heterogeneity in the agent's valuation of the interaction on the platform.

and online intermediation services.[5]

The perhaps the most influential article on competition in two-sided markets with these features is Armstrong (2006). He considers competition between two intermediaries in a Hotelling model of horizontal differentiation. Agents within a group are assumed to be heterogeneous in their preferences over platform types. He also assumes that an agent's benefit from interacting with agents on the other side of the platform is independent of platform type, and that this benefit is invariant between agents within the same group.

Most previous studies on horizontal mergers in two-sided markets consider a setting where different groups of consumers interact through a platform and where platforms offer horizontally differentiated services. The literature on the theoretical welfare effects of horizontal mergers (henceforth only 'mergers') in two-sided markets, is quite scarce and the focus in theoretical as well as empirical work has often been on mediamarkets and in particular the newspaper industry. There are a number of empirical studies on mergers in two-sided markets that consider these market-types including Filistrucchi et al. (2012), Jeziorski (2014) and Van Cayseele & Vanormelingen (2019).[6]

An early and somewhat specific contribution to the theoretical literature on mergers in two-sided markets is Chandra & Collard-Wexler (2008). The authors consider a model of two competing newspapers and two advertisers of differentiated products. Readers are distributed over the Hotelling line and it is assumed that readers' demand for products and newspapers are correlated. Key to their result is their assumption of below marginal cost pricing on the reader side of the market. The advertisers have low willingness to pay for the attention of marginal readers. Hence, the revenue generated by marginal readers may be insufficient to cover the subsidy they get by the newspaper. As a result, the duopoly equilibrium may imply higher prices for both sides of the market than the prices set by a monopolist that internal-

[5]See Rochet & Tirole (2003) for a more examples of traditional industries as well as new economy industries with these features.

[6]Filistrucchi et al. (2012) study the Dutch newspaper market, Van Cayseele & Vanormelingen (2019) consider the Belgian newspaper industry and Jeziorski (2014) estimates the effects of a merger in the US radio industry.

izes this effect.[7]

Mergers in two-sided markets are analyzed in a similar fashion in Leonello (2011). Her model is particularly well suited for the newspaper industry although the analysis applies more generally to other settings.[8] She considers a case where two competing platforms merge into a joint ownership with some degree of interoperability between the platforms. Specifically, the merged entity offers consumers the possibility to interact with users on both platforms. It is shown that the merger can be welfare enhancing even absent efficiency gains if the indirect network effects are sufficiently strong. Cosnita-Langlais et al. (2016) extend her analysis by considering competition between four adjacent platforms in a spatial model à la Salop. The authors assume perfectly symmetrical groups of agents and that a merger results in marginal cost reductions for the platforms that merge. The authors show that a merger may lead to lower prices in the presence of efficiency gains if network effects are sufficiently strong.

Chandra & Collard-Wexler (2008), Leonello (2011) and Cosnita-Langlais et al. (2016) all assume that a merger between two platforms implies joint ownership of the two merging platforms. Correia-da-Silva et al. (2019) consider the case where the number of active platforms is reduced as a result of a merger. Specifically, they consider a merger in a two-sided market where $k \in \{1, ..., K\}$ homogeneous platforms compete à la Cournot. The extent to which agents on one side benefit from the interaction with agents on the other side of a platform depends on which of the platforms that is joined. As a result, the platforms charge different prices but the same externality adjusted prices in equilibrium.[9] It is shown that consumers on both sides of the market benefit from a merger if the pre-merger externality adjusted prices are below marginal costs on both sides of the market.

[7] Chandra & Collard-Wexler (2008) find empirical evidence that supports this result by using data from the Canadian newspaper industry.

[8] For instance, she assumes that only one side of the market obtains benefits from the interaction with agents of the other side and that both groups are symmetrical with respect to their preferences for platform types.

[9] Platform $k$ charges price $p_i^k$ to side $i \in \{1, 2\}$ of the market. The externality adjusted price is defined as $p_i^k - \alpha_i n_{-i}^k$ where $\alpha_i$ is the benefit side $i$ obtain from interacting with agents on the other side of the platform and $n_{-i}$ is the number of agents on the other side of the platform.

The choice of the merged entity to either shut down one platform or keep both is not investigated in the above-mentioned papers, but rather assumed to be exogenously determined. This issue is in part considered in Tan & Zhou (2019). They provide a more general theoretical framework by considering competition between multiple platforms that serve multiple groups of agents. In an illustration of a merger between two platforms in a market constituted of three competing platforms they find that the joint profit of two merged platforms is higher than the individual profits in the duopoly setting. Thereby they conclude that a merged platform has no incentives to shut down one platform and compete with the remaining one.

Similar to Chandra & Collard-Wexler (2008) and Leonello (2011) I consider a merger between two competing platforms located at the endpoints of a Hotelling line and assume that consumers patronize only one platform. While the focus in their studies is on the newspaper market, I consider a merger between two digital platforms more generally. In particular I do not assume any specific relation between the preferences of the consumer groups or between the consumer groups' benefits from interacting on a platform.[10] In addition I investigate the choice of the merged entity to shut down one platform or keep both under a joint ownership. In contrast to Tan & Zhou (2019) I assume that platforms have fixed costs and that fixed costs are higher for a merged entity that operates two platforms.

[10] The analysis in Chandra & Collard-Wexler (2008) is specifically adapted to the conditions in the newspaper market. Leonello (2011) considers the case where consumer groups have equal preferences and where only one group benefit from the interaction.

# Part I

# Duopoly in a two-sided market

## 3 A model of two competing platforms

The analysis follows a model of competing intermediaries (or platforms) presented in section 4 in Armstrong (2006). For illustrative purposes, the model will here be presented in the light of a situation where buyers and sellers interact on the platforms. However, the model applies more generally to other settings where agents of some group interact with agents of some other group.

Consider two differentiated platforms that compete for buyers and sellers. There are two sides of the market, a buyer and a seller side that have heterogeneous preferences over the two platform types. Specifically, members of both groups are uniformly and continuously distributed over the unit interval according to their preferences over platform types. The two platforms are located at the endpoints of the interval. To facilitate notation, let the competing platforms be denoted as platform 0 and platform 1 respectively in accordance with their location in the interval. Indirect network effects are present, so a buyer's valuation of a platform increases in the number of sellers on the platform and vice versa.

The situation is modelled as a two-stage game. At the first stage the competing platforms set their prices simultaneously. At the second stage the buyers and sellers make a choice to join one of the two platforms, conditional on the prices set by the platforms and on their respective expectations about the number of participants on the other side of the platforms. In equilibrium, the expectations of the buyers and sellers are consistent.

Let $p_b^i$ and $p_s^i$ denote the fixed fees that buyers and sellers are charged respectively by platform $i \in \{0, 1\}$ to join that platform. Let $n_s^i$ and $n_b^i$ be the respective number of sellers and buyers that use platform $i$. The buyer and seller utilities on platform $i$ (gross of any disutility from not being able to use the most preferred platform type) that attracts $n_b^i$ buyers and $n_s^i$ sellers are given by:

$$v_b^i = n_s^i \alpha_b - p_b^i \tag{1}$$

$$v_s^i = n_b^i \alpha_s - p_s^i \tag{2}$$

where parameter $\alpha_b > 0$ denotes the benefit a buyer obtains from interacting with a seller on the same platform and similarly, parameter $\alpha_s > 0$ is the benefit a seller obtains from interacting with a buyer on that platform. The parameters $\alpha_b$ and $\alpha_s$ thus measure the indirect network effects for buyers and sellers and are independent of which platform the groups interact on. Note that these specifications imply that consumers of one group only value a platform by its ability to enable the interaction with consumers of the other group and that platforms are perceived as homogeneous with respect to their provision of interaction possibilities.

Buyers and sellers incur a transportation cost when joining a platform that increases linearly in their distance to the platform at rate $\tau_b > 0$ and $\tau_s > 0$ respectively. This transportation cost is to be interpreted as the buyers' and sellers' respective disutility from not being able to choose their most preferred platform type, and is a measure of the perceived horizontal differentiation between the two platforms. Since platforms are perceived as homogeneous in their provision of interaction possibilities, horizontal differentiation can in this context be reflected in differences in layout or other features that affects the platform experience but are insignificant for the intrinsic benefit of the interaction.

Following Armstrong's (2006) specification of user gross utilities on the platforms given in (1) and (2), the net utility for a buyer located at $x \in [0, 1]$ from using platform 0 and 1 is given by $U_b^0(x) = v_b^0 - x\tau_b$ and $U_b^1(x) = v_b^1 - (1-x)\tau_b$ respectively. Analogously, the net utility of a seller located at $y \in [0, 1]$ from using platform 0 and 1 is given by $U_s^0(y) = v_s^0 - y\tau_s$ and $U_s^1(y) = v_s^1 - (1-y)\tau_s$ respectively.

Users participate in the market whenever the net utility (henceforth only 'utility') from joining a platform is non-negative. If both platforms yield non-negative utility then users will join the platform that generates greater utility. Provided that every buyer and seller is active in the market, there is a buyer and a seller in $[0, 1]$ that is indifferent between joining platform 0 or 1. These agents get the lowest utility from participating in the market and it follows that the market is covered if their utility from joining a platform is non-negative. Armstrong (2006) assumes that every buyer and seller obtains a sufficiently high level of utility from participating in the market to ensure that the market is covered. The total number of buyers is then $n_b^0 + n_b^1 = 1$ and the total number of sellers is given by $n_s^0 + n_s^1 = 1$.

Note that a covered market implies that the platforms share a segment of the market where they compete for buyers and sellers. In the opposite case where parameter values are such that the indifferent buyer and seller obtain negative utility and thereby abstain from market participation, the platforms enjoy positions as local monopolists. From a competition policy perspective, this is not a relevant case to consider since the lack of strategic interaction between the platforms implies that the equilibrium would not be altered as a result of a merger.

Now, let the indifferent buyer be located at $\hat{x}$. Then:

$$\hat{U}_b^0 \equiv U_b^0(\hat{x}) = v_b^0 - \hat{x}\tau_b = v_b^1 - (1-\hat{x})\tau_b = U_b^1(\hat{x}) \equiv \hat{U}_b^1$$

$$\Leftrightarrow \hat{x} = \frac{1}{2} + \frac{v_b^0 - v_b^1}{2\tau_b} \tag{3}$$

Correspondingly, the indifferent seller is located at:

$$\hat{y} = \frac{1}{2} + \frac{v_s^0 - v_s^1}{2\tau_s} \tag{4}$$

The market is covered if the indifferent buyer and seller obtain non-negative net utility from market participation. Hence, every buyer participate in the market if $U_b^0(\hat{x}) = U_b^1(\hat{x}) \equiv \hat{U}_b \geq 0$. Similarly, all the sellers participate in the market if $U_s^0(\hat{y}) = U_s^1(\hat{y}) \equiv \hat{U}_s \geq 0$.

It follows that the market is covered if:

$$\hat{U}_b = v_b^0 - \hat{x}\tau_b = v_b^1 - (1-\hat{x})\tau_b \geq 0 \tag{5}$$

$$\hat{U}_s = v_s^0 - \hat{y}\tau_s = v_s^1 - (1-\hat{y})\tau_s \geq 0 \tag{6}$$

Substituting for $\hat{x}$ and $\hat{y}$ in (5) and (6) yields:

$$\hat{U}_b = \frac{v_b^0 + v_b^1 - \tau_b}{2} \geq 0 \tag{7}$$

$$\hat{U}_s = \frac{v_s^0 + v_s^1 - \tau_s}{2} \geq 0 \tag{8}$$

Now, using the expressions given in (1) and (2) in the inequalities in (7) and (8), and using that $n_b^0 + n_b^1 = 1$ and $n_s^0 + n_s^1 = 1$ gives:

$$\hat{U}_b = \frac{n_s^0\alpha_b - p_b^0 + n_s^1\alpha_b - p_b^1 - \tau_b}{2} = \frac{\alpha_b - (p_b^0 + p_b^1) - \tau_b}{2} \geq 0$$
$$\hat{U}_s = \frac{n_b^0\alpha_s - p_s^0 + n_b^1\alpha_s - p_s^1 - \tau_s}{2} = \frac{\alpha_s - (p_s^0 + p_s^1) - \tau_s}{2} \geq 0$$

It follows that, the market is covered when the parameter values are such that the full market participation constraints, FMPCB and FMPCS, are satisfied:

$$\alpha_b - (p_b^0 + p_b^1) - \tau_b \geq 0 \tag{FMPCB}$$

$$\alpha_s - (p_s^0 + p_s^1) - \tau_s \geq 0 \tag{FMPCS}$$

If this is the case, then every buyer located in $[0, \hat{x}]$ chooses to join platform 0, and every buyer located in $(\hat{x}, 1]$ chooses to join platform 1. The demand for platform 0 (equivalent to $n_b^0$, the number of buyers that use platform 0) is thus given by $n_b^0 = \hat{x} = \frac{1}{2} + \frac{v_b^0 - v_b^1}{2\tau_b}$ and the demand for platform 1 (i.e. $n_b^1$, number of buyers that use platform 1) is given by $n_b^1 = 1 - \hat{x} = \frac{1}{2} + \frac{v_b^1 - v_b^0}{2\tau_b}$. Analogously, the number of sellers that joins platform 0 is given by $n_s^0 = \hat{y} = \frac{1}{2} + \frac{v_s^0 - v_s^1}{2\tau_s}$, and the number of sellers that joins platform 1 is given by $n_s^1 = 1 - \hat{y} = \frac{1}{2} + \frac{v_s^1 - v_s^0}{2\tau_s}$.

Hence, the number of buyers and sellers that chooses to join platform $i$ is given by:

$$n_b^i = \frac{1}{2} + \frac{v_b^i - v_b^{-i}}{2\tau_b} \tag{9}$$
$$n_s^i = \frac{1}{2} + \frac{v_s^i - v_s^{-i}}{2\tau_s} \tag{10}$$

We can now derive the respective demands for platform $i$ for buyers and sellers, conditional on their respective expectations of the number of participants on the other side of the same platform. Using (1) and (2) and the fact that $n_b^i = 1 - n_b^{-i}$ and $n_s^i = 1 - n_s^{-i}$ in (9) and (10) yields:

$$n_b^i(n_s^i) = \frac{1}{2} + \frac{1}{2\tau_b}[(2n_s^i - 1)\alpha_b - (p_b^i - p_b^{-i})] \tag{11}$$
$$n_s^i(n_b^i) = \frac{1}{2} + \frac{1}{2\tau_s}[(2n_b^i - 1)\alpha_s - (p_s^i - p_s^{-i})] \tag{12}$$

From (11) and (12) it can be noted that the buyers' demand for platform $i$ depends on the expected number of sellers on platform $i$ and correspondingly, the sellers' demand for platform $i$ depends on the expected number of buyers on that platform. We notice that a market sharing equilibrium requires some degree of product differentiation. In the extreme case with homogeneous platforms ($\tau_b \to 0$ and $\tau_s \to 0$) the ratios $\frac{1}{2\tau_b}$ and $\frac{1}{2\tau_s}$ become infinitely large. As has been concluded by Caillaud & Jullien (2003), the only equilibrium in a setting with homogeneous platforms is that one platform attracts all agents.

By solving the equation system (11) and (12) we get the expressions for the consistent expectations equilibrium number of buyers and sellers:

$$n_b^i = \frac{1}{2} + \frac{\alpha_b(p_s^{-i} - p_s^i) + \tau_s(p_b^{-i} - p_b^i)}{2(\tau_b\tau_s - \alpha_b\alpha_s)} \tag{13}$$

$$n_s^i = \frac{1}{2} + \frac{\alpha_s(p_b^{-i} - p_b^i) + \tau_b(p_s^{-i} - p_s^i)}{2(\tau_b\tau_s - \alpha_b\alpha_s)} \tag{14}$$

When the indirect network effects are sufficiently weak in relation to the degree of product differentiation, such that $\tau_b\tau_s > \alpha_b\alpha_s$, the number of buyers and sellers on a given platform are decreasing functions of the user fee charged by that platform. The network size is then monotonically decreasing in prices and there is a unique demand level at any given price. The condition that $\tau_b\tau_s > \alpha_b\alpha_s$ ensures that two platforms can be active in the same market, and will henceforth be referred to as the Market Sharing Condition (MSC):[11]

$$\tau_b\tau_s > \alpha_b\alpha_s \tag{MSC}$$

Assuming that platform $i$ has constant marginal cost, $c_b$ and $c_s$, of providing the service to buyers and sellers we get the following expression for the profit of platform $i$:

$$\Pi^i = (p_s^i - c_s)n_s^i + (p_b^i - c_b)n_b^i \tag{15}$$

[11] When agents have heterogeneous preferences for the platform type and there are strong network effects, such that $\tau_b\tau_s < \alpha_b\alpha_s$, the number of participants on each side of the market is an increasing function of the price: $n_b^i = \frac{1}{2} + \frac{\alpha_b(p_s^i - p_s^j) + \tau_s(p_b^i - p_b^j)}{2(\alpha_b\alpha_s - \tau_b\tau_s)}$, and $n_s^i = \frac{1}{2} + \frac{\alpha_s(p_b^i - p_b^j) + \tau_b(p_s^i - p_s^j)}{2(\alpha_b\alpha_s - \tau_b\tau_s)}$. There are then multiple equilibrium network sizes and the interior equilibrium, where $n_b^i, n_s^i \in [0,1]$, is unstable, whereas $n_b^i, n_s^i = 0$ and $n_b^i, n_s^i = 1$ are stable equilibria. This creates tipping effects and a situation where one of the platforms in the market attracts all the users (Belleflamme & Peitz 2010).

The competing platforms anticipate the buyers and sellers behavior and choose their prices simultaneously. Thus, platform $i$ solves the following problem:

$$\max_{p_b^i, p_s^i} \Pi^i = (p_s^i - c_s)\Big(\frac{1}{2} + \frac{\alpha_s(p_b^{-i} - p_b^i) + \tau_b(p_s^{-i} - p_s^i)}{2(\tau_b\tau_s - \alpha_b\alpha_s)}\Big) + (p_b^i - c_b)\Big(\frac{1}{2} + \frac{\alpha_b(p_s^{-i} - p_s^i) + \tau_s(p_b^{-i} - p_b^i)}{2(\tau_b\tau_s - \alpha_b\alpha_s)}\Big) \quad (16)$$

The first order conditions are:

$$\frac{\partial \Pi^i}{\partial p_s^i} = \Big(\frac{1}{2} + \frac{\alpha_s(p_b^{-i} - p_b^i) + \tau_b(p_s^{-i} - p_s^i)}{2(\tau_b\tau_s - \alpha_b\alpha_s)}\Big) - \frac{\tau_b(p_s^i - c_s)}{2(\tau_b\tau_s - \alpha_b\alpha_s)} - \frac{\alpha_b(p_b^i - c_b)}{2(\tau_b\tau_s - \alpha_b\alpha_s)} = 0$$
$$\iff \frac{1}{2} + \frac{\alpha_s p_b^{-i} - p_b^i(\alpha_s + \alpha_b) + \tau_b(p_s^{-i} - 2p_s^i + c_s) + \alpha_b c_b}{2(\tau_b\tau_s - \alpha_b\alpha_s)} = 0 \quad (17)$$

$$\frac{\partial \Pi^i}{\partial p_b^i} = \Big(\frac{1}{2} + \frac{\alpha_b(p_s^{-i} - p_s^i) + \tau_s(p_b^{-i} - p_b^i)}{2(\tau_b\tau_s - \alpha_b\alpha_s)}\Big) - \frac{\tau_s(p_b^i - c_b)}{2(\tau_b\tau_s - \alpha_b\alpha_s)} - \frac{\alpha_s(p_s^i - c_s)}{2(\tau_b\tau_s - \alpha_b\alpha_s)} = 0$$
$$\iff \frac{1}{2} + \frac{\alpha_b p_s^{-i} - p_s^i(\alpha_b + \alpha_s) + \tau_s(p_b^{-i} - 2p_b^i + c_b) + \alpha_s c_s}{2(\tau_b\tau_s - \alpha_b\alpha_s)} = 0 \quad (18)$$

The second order conditions are:

$$\frac{\partial^2 \Pi^i}{\partial {p_b^i}^2} = -\frac{\tau_s}{\tau_b\tau_s - \alpha_b\alpha_s} < 0 \quad (19)$$
$$\frac{\partial^2 \Pi^i}{\partial {p_s^i}^2} = -\frac{\tau_b}{\tau_b\tau_s - \alpha_b\alpha_s} < 0 \quad (20)$$

$$|\mathbf{H}(p_b^i, p_s^i)| = \frac{\partial^2 \Pi^i}{\partial {p_b^i}^2} \cdot \frac{\partial^2 \Pi^i}{\partial {p_s^i}^2} - \Big[\frac{\partial^2 \Pi^i}{\partial p_b^i \partial p_s^i}\Big]^2 = \frac{\tau_s\tau_b}{(\tau_b\tau_s - \alpha_b\alpha_s)^2} - \frac{(\alpha_b + \alpha_s)^2}{4(\tau_b\tau_s - \alpha_b\alpha_s)^2} = \frac{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}{4(\tau_b\tau_s - \alpha_b\alpha_s)^2} > 0 \quad (21)$$

Conditions (19) and (20) hold by the Market Sharing Condition, (MSC). Condition (21) is satisfied if, and only if:

$$4\tau_b\tau_s > (\alpha_b + \alpha_s)^2 \tag{EC}$$

This is a stricter restriction on the indirect network effects than (MSC). It follows that it is necessary and sufficient that (EC) is satisfied for (MSC) and the second order conditions for a maximum to hold, and this condition is therefore referred to as the Equilibrium Condition (EC).[12]

If (EC) holds, platform $i$'s profit maximizing choice of prices $(p_b^i, p_s^i)$, given the combination of prices set by the competing platform, is an interior critical point for platform $i$'s profit function. Platform $i$'s best response to its competitor's prices can then be solved from the system of first order equations (17) and (18).

By imposing symmetry, $p_b^i = p_b^{-i} = p_b$ and $p_s^i = p_s^{-i} = p_s$, Armstrong (2006) finds that:

$$\frac{1}{2} + \frac{-\alpha_b p_b - \tau_b p_s + \tau_b c_s + \alpha_b c_b}{2(\tau_b\tau_s - \alpha_b\alpha_s)} = 0 \iff p_s = c_s + \tau_s - \frac{\alpha_b}{\tau_b}(\alpha_s + p_b - c_b) \tag{22}$$

$$\frac{1}{2} + \frac{-\alpha_s p_s - \tau_s p_b + \tau_s c_b + \alpha_s c_s}{2(\tau_b\tau_s - \alpha_b\alpha_s)} = 0 \iff p_b = c_b + \tau_b - \frac{\alpha_s}{\tau_s}(\alpha_b + p_s - c_s) \tag{23}$$

Solving the system of first order equations yields:

$$p_s = c_s + \tau_s - \alpha_b \tag{24}$$

$$p_b = c_b + \tau_b - \alpha_s \tag{25}$$

Now, the equations in (13) and (14) reduce to:

$$n_b^i = \frac{1}{2} \tag{26}$$

$$n_s^i = \frac{1}{2} \tag{27}$$

[12](EC) corresponds to assumption 8 in Armstrong (2006) and is assumed to hold throughout the analysis.

In a symmetric equilibrium each platform will serve an equal share of buyers and sellers.

The derived equilibrium is consistent with the assumption that the platforms share a market segment if the parameter values are such that the market is covered in equilibrium. Recall the constraints for a covered market, (FMPCB) and (FMPCS). By inserting the equilibrium prices set by the platforms into these expressions we obtain the restrictions on the parameters such that the market is covered in equilibrium:

$$\alpha_b + 2\alpha_s - 3\tau_b - 2c_b \geq 0 \tag{FMPCB*}$$

$$\alpha_s + 2\alpha_b - 3\tau_s - 2c_s \geq 0 \tag{FMPCS*}$$

Hence, conditions (FMPCB*) and (FMPCS*), together with (EC) must be satisfied in this model. By solving these conditions for $\alpha_s$ we get:[13]

$$\alpha_s \geq \frac{3}{2}\tau_b + c_b - \frac{1}{2}\alpha_b \tag{28}$$

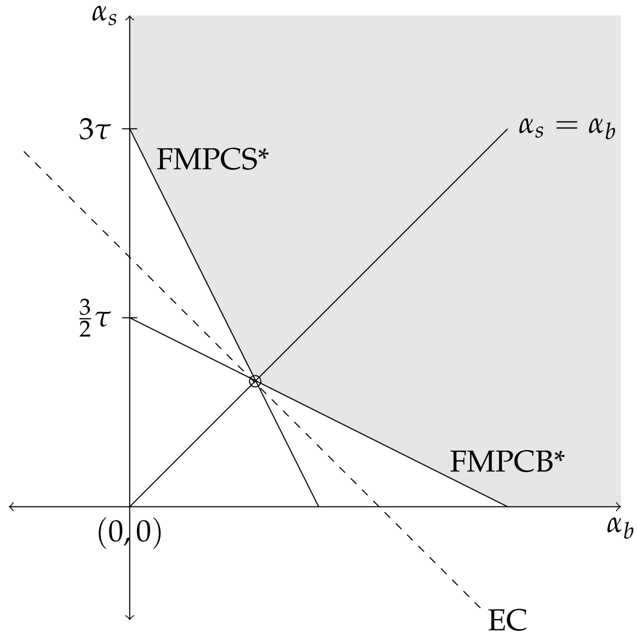
$$\alpha_s \geq 3\tau_s + 2c_s - 2\alpha_b \tag{29}$$

$$\alpha_s < 2\sqrt{\tau_b \tau_s} - \alpha_b \tag{30}$$

For fixed values of $\tau_b$, $\tau_s$, $c_b$ and $c_s$, the conditions can be illustrated graphically in $(\alpha_b, \alpha_s)$-plane. In Figure 1 below, $\tau_b$ and $\tau_s$ are set to $\tau_b = \tau_s = \tau$, where $\tau$ is some positive constant, and $c_b$ and $c_s$ are set equal to 0.

[13] The inequality in (30) is equivalent to (EC) since $\alpha_b, \alpha_s, \tau_b, \tau_s > 0$.

Figure 1




The shaded area below the dashed line represent the set of combinations of $(\alpha_b, \alpha_s)$ for which the Equilibrium Condition (EC) is satisfied. The light shaded area above the full market participation constraints for buyers and sellers represents the set of combinations of $(\alpha_b, \alpha_s)$ for which the market is covered. Notice that any pair $(\alpha_b, \alpha_s)$ that satisfies (EC) is incompatible with a covered market.

When assuming that marginal costs for serving both sides of the markets are positive, the intercept of the full market participation constraints of buyers and sellers increase. The result is an upward shift in these constraints, further away from the set of combinations of $(\alpha_b, \alpha_s)$ for which (EC) is satisfied.

The presentation in Figure 1 assumes that buyers and sellers are equal with respect to their preferences for platform type. Nonetheless, it can be shown that there exist no parameter values such that the conditions for a covered market in equilibrium, (FMPCB*) and (FMPCS*); and the necessary and sufficient condition for a unique market sharing equilibrium, (EC); are compatible. The result is summarized in Proposition 1.

**Proposition 1** *The second order conditions for an interior solution in the model proposed* *by Armstrong (2006) are not compatible with the assumption of a covered market in equilibrium.*

*Proof.* See appendix A.1. ■

The result is not surprising. Firstly, in case consumers of one group only value a platform by its ability to enable the interaction with consumers of the other group, then indirect network effects must be sufficiently high in relation to the degree of platform differentiation to ensure that the platforms' share market areas. Second, consumers perceive platforms as homogeneous in their provision of interaction possibilities. As mentioned in the beginning of this section, horizontal differentiation can in this context be reflected in differences in for example layout or similar features. There is no reason to assume that consumers of any group have very strong preferences for one platform or the other in such contexts. This conflicts with the requirement that the degree of platform differentiation must be sufficiently high to ensure a unique market sharing equilibrium.

The model presented by Armstrong (2006) is then not very useful for a merger analysis. One way to ensure that the second order conditions for an interior solution are compatible with the conditions for a covered market is to assume that buyers or sellers obtain some benefit from using a platform independently of the number of agents on the other side of the platform. This is motivated when platforms provide additional content that consumers value. While this precludes an analysis of mergers in markets that lack such features like most e-commerce marketplaces, there are many other digital platform markets where the assumption fits. Examples include digital newspapers and magazines, online advertising platforms (such as blogs and online streaming markets) or platforms providing intermediation services as well as information to users (consider for example online travel intermediaries that provide users with information on travel destinations while also enabling the interaction between travellers and accommodation providers).

In the following section I extend the buyers' and sellers' respective gross utilities from using a platform with a constant benefit representing users' valuation of the intrinsic quality of a platform. It is shown that when such stand-alone benefits are present, the benefits from interacting on a platform can be lower and still ensure full market participation. In this case, the conditions for a covered market and the second order conditions for an interior solution can hold simultaneously.

# 4 Introducing stand-alone benefits

Consider the model presented in the previous section, but now suppose that buyers and sellers also obtain a constant, non-negative, stand-alone benefit from participating on a platform, and that this benefit is independent of the number of participants on the other side of the platform. This is to be interpreted as the benefit users obtain by the platforms' provision of some additional content.

Letting a user's stand-alone benefit from using a platform be independent of which platform is joined, the gross utilities (1) and (2) on platform $i$ expands to:

$$w_b^i = s_b + n_s^i \alpha_b - p_b^i \tag{31}$$

$$w_s^i = s_s + n_b^i \alpha_s - p_s^i \tag{32}$$

where $s_b \geq 0$ and $s_s \geq 0$ denote the stand-alone benefit that a buyer and a seller derive respectively from joining any of the two platforms.

Recall that buyers and sellers are continuously and uniformly distributed over the unit interval according to their preferences over platform types and that the two competing platforms are located at the extreme points of the interval. In this context, platforms may be differentiated in terms of how the provided content appeals to consumers. Newspapers that serve readers and advertisers may for example differ in terms of political position, travel intermediaries may provide information on different types of travel destinations and blogs can be differentiated in terms of content and thereby attract different types of readers and advertisers.

A buyer and a seller will choose to join the platform that yields the highest utility. As before, the indifferent buyer and seller is located at $\hat{x}$ and $\hat{y}$ respectively. By the modified specifications of the gross utilities it follows that the respective location of the indifferent buyer and seller is given by:

$$\hat{x} = \frac{1}{2} + \frac{w_b^0 - w_b^1}{2\tau_b} \tag{33}$$

$$\hat{y} = \frac{1}{2} + \frac{w_s^0 - w_s^1}{2\tau_s} \tag{34}$$

As long as every buyer derives non-negative utility from joining a platform, every buyer located at $x \in [0, \hat{x}]$ chooses to join platform 0, and every buyer located at $x \in (\hat{x}, 1]$ joins platform 1. Similarly, when every seller obtains non-negative utility from participation, then all sellers located at $y \in [0, \hat{y}]$ choose to join platform 0, and every seller located at $(\hat{y}, 1]$ joins platform 1. The number of buyers and sellers that join platform $i$ is then given by:

$$n_b^i = \frac{1}{2} + \frac{w_b^i - w_b^{-i}}{2\tau_b} \tag{35}$$

$$n_s^i = \frac{1}{2} + \frac{w_s^i - w_s^{-i}}{2\tau_s} \tag{36}$$

Now using (31) and (32) in (35) and (36):

$$n_b^i = \frac{1}{2} + \frac{n_s^i \alpha_b - p_b^i - (n_s^{-i}\alpha_b - p_b^{-i})}{2\tau_b} = \frac{1}{2} + \frac{v_b^i - v_b^{-i}}{2\tau_b} \tag{37}$$

$$n_s^i = \frac{1}{2} + \frac{n_b^i \alpha_s - p_s^i - (n_b^{-i}\alpha_s - p_s^{-i})}{2\tau_s} = \frac{1}{2} + \frac{v_s^i - v_s^{-i}}{2\tau_s} \tag{38}$$

where the last equalities follow from the definitions in (1) and (2). Note that (37) and (38) are identical to (9) and (10). As a consequence, the subsequent steps in the derivation of the consistent expectation equilibrium network sizes and the equilibrium duopoly prices will be identical to the corresponding steps in section 3. Hence, when the gross utilities on the platform are extended with stand-alone benefits the equilibrium duopoly prices and network sizes will nevertheless be given by (24), (25), (26) and (27), repeated here for the convenience of the reader:

$$p_s = c_s + \tau_s - \alpha_b$$

$$p_b = c_b + \tau_b - \alpha_s$$

$$n_b^i = \frac{1}{2}$$

$$n_s^i = \frac{1}{2}$$

In section 3 it was established that the equilibrium is characterized by these equations when the market sharing condition and the second order conditions for a maximum are both satisfied and that this is the case if and only if the Equilibrium Condition (EC) hold. It should be evident that this applies also in the case with extended gross utilities.

So far, the analysis based on the gross utilities in (31) and (32) has been identical to the analysis in section 3. What is different is the utility of the indifferent buyer and seller. Recall that platforms have an overlapping market area where they compete for buyers and sellers only if the market is covered and that this is the case when the indifferent buyer and seller obtain non-negative utility from joining a platform:

$$\hat{U}_b = w_b^0 - \hat{x}\tau_b = w_b^1 - (1-\hat{x})\tau_b \geq 0 \tag{39}$$

$$\hat{U}_s = w_s^0 - \hat{y}\tau_s = w_s^1 - (1-\hat{y})\tau_s \geq 0 \tag{40}$$

We can use the expressions for the location of the indifferent buyer and seller, (33) and (34), into the expressions above:

$$\hat{U}_b = \frac{w_b^0 + w_b^1 - \tau_b}{2} \geq 0 \tag{41}$$

$$\hat{U}_s = \frac{w_s^0 + w_s^1 - \tau_s}{2} \geq 0 \tag{42}$$

Using (31) and (32) and the fact that $n_b^0 + n_b^1 = 1$ and $n_s^0 + n_s^1 = 1$ in (41) and (42) yields:

$$\hat{U}_b = \frac{\alpha_b - (p_b^0 + p_b^1) - \tau_b + 2s_b}{2} \geq 0 \tag{43}$$

$$\hat{U}_s = \frac{\alpha_s - (p_s^0 + p_s^1) - \tau_s + 2s_s}{2} \geq 0 \tag{44}$$

Hence, a covered market implies that the extended full market participation constraints are satisfied for buyers and sellers respectively:

$$\alpha_b - (p_b^0 + p_b^1) - \tau_b + 2s_b \geq 0 \tag{FMPCB2}$$

$$\alpha_s - (p_s^0 + p_s^1) - \tau_s + 2s_s \geq 0 \tag{FMPCS2}$$

Inserting the equilibrium prices and the corresponding network sizes into the full market participation constraints (FMPCB2) and (FMPCS2) gives:

$$\alpha_b + 2\alpha_s - 3\tau_b + 2(s_b - c_b) \geq 0 \tag{FMPCB2*}$$

$$\alpha_s + 2\alpha_b - 3\tau_s + 2(s_s - c_s) \geq 0 \tag{FMPCS2*}$$

It should be evident that, unless *both* $s_b$ and $s_s$ are equal to 0 the restrictions on the parameters such that the market is covered in equilibrium given by (FMPCB2*) and (FMPCS2*) are less restrictive than the corresponding conditions in section 3: (FMPCB*) and (FMPCS*). Hence, if stand-alone benefits are sufficiently large for at least one side of the market, there exist parameter values that satisfy the conditions for a covered market in equilibrium, (FMPCB2*) and (FMPCS2*), and the necessary and sufficient condition for a unique market sharing equilibrium, (EC).

The intuition is that if stand-alone benefits from using a platform are sufficiently high for at least one of the groups, the indirect network effect can be lower for that group and still ensure full market participation. The upper restrictions on the indirect network effects given by (EC) can then be consistent with full market participation for both groups.

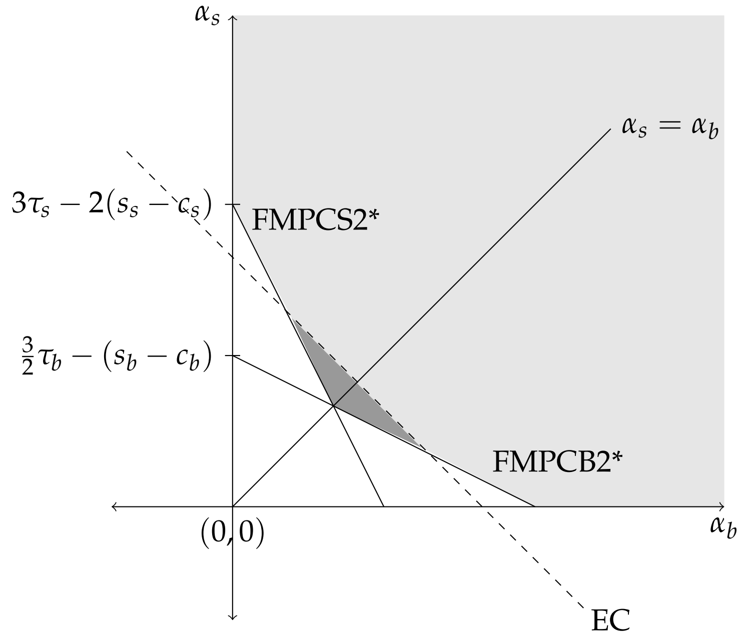
For fixed values of $\tau_b$, $\tau_s$, $c_b$, $c_s$, $s_b$ and $s_s$, the conditions (FMPCB2*), (FMPCS2*) and (EC), can be illustrated graphically in $(\alpha_b, \alpha_s)$-plane. Solving for $\alpha_s$ in (FMPCB2*) and (FMPCS2*) yields:

$$\alpha_s \geq \frac{3}{2}\tau_b - (s_b - c_b) - \frac{1}{2}\alpha_b \tag{45}$$

$$\alpha_s \geq 3\tau_s - 2(s_s - c_s) - 2\alpha_b \tag{46}$$

Figure 2 below illustrates the conditions in $(\alpha_b, \alpha_s)$-plane in the case when buyers and sellers are equally differentiated and the stand-alone benefits are higher than marginal costs for both sides of the market:

Figure 2

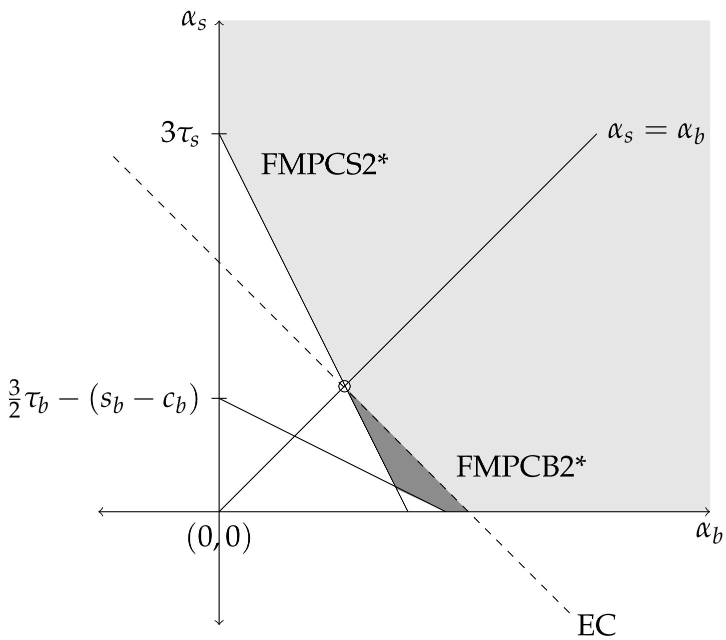



The shaded area below the dashed line is the set of combinations of $(\alpha_b, \alpha_s)$ for which condition (EC) is satisfied; and the shaded area above the full market participation constraints is the set of pairs $(\alpha_b, \alpha_s)$ for which the market is covered. From Figure 2 it can be observed that sufficiently high positive stand-alone benefits decrease the intercepts of the full market participation constraints of buyers and sellers and there is an overlapping segment where the full market participation constraints and condition (EC) are satisfied simultaneously. The darkest shaded area in the figure represent the set of combinations of $(\alpha_b, \alpha_s)$ for which all conditions are satisfied for fixed parameter values.

Now suppose instead that only one side of the market, say buyers, obtain a stand-alone benefit from using the platform. Again, we can illustrate the conditions for a covered market together with the equilibrium condition in $(\alpha_b, \alpha_s)$-plane for fixed values of the other parameters.

Figure 3 below illustrates the case where buyers and sellers are equally differentiated, stand-alone benefits for sellers and the marginal costs of serving the seller side of the market are zero, and the stand-alone benefit for buyers exceeds the marginal cost of serving the buyer side of the market.

Figure 3: $\tau_b = \tau_s$




When buyers obtain sufficiently high stand-alone benefits from joining a platform, the intercept of the full market participation constraint for buyers decreases. For any given value of $\alpha_s$, the benefit buyers obtain from interacting on a platform, $\alpha_b$, can now be lower and still ensure that every buyer participate in the market.

There is an overlapping segment, represented by the darkest shaded area, where all conditions are satisfied. Interestingly, the conditions are all satisfied for parameter values such that $\alpha_b > \alpha_s$ in the case that buyers and sellers are equally differentiated.

We conclude the following.

**Proposition 2** *Sufficiently large stand-alone benefits for at least one side of the market are necessary to ensure that the second order conditions for an interior solution are compatible with the conditions for a covered market.*

*In case only one group $k \in \{b, s\}$ obtains stand-alone benefits from joining a platform the following conditions must hold:*

$$s_k > \frac{3}{2}(\sqrt{\tau_b} - \sqrt{\tau_s})^2 + c_b + c_s \tag{47}$$

$$3\tau_{-k} + 2c_{-k} - 4\sqrt{\tau_k \tau_{-k}} < 0 \tag{48}$$

*Proof.* See appendix A.2 ■

Having established that the equilibrium is consistent with the assumption of a covered market in the presence of stand-alone benefits we can now proceed with the analysis of a merger in markets where such benefits are present.

# Part II

# A horizontal merger

Suppose that the two competing platforms described in the previous section merge into a monopoly. I will assume that a merger between the two firms does not lead to a reduction in marginal costs of serving the two sides of the market for any of the platform types, 0 and 1. Any gains from the merger are assumed to be due to reductions in fixed costs. These presumptions are motivated by the fact that fixed costs are typically high while marginal costs are typically low in digital platform markets. It is therefore reasonable to assume that the effect on marginal costs from a merger are negligible and that any gains are motivated by savings in fixed costs.

## 5 The choices of the merged entity

If the two firms merge the market becomes a monopoly. I assume that platforms' location do not change as a result of a merger which implies that platforms continue to offer the same services. Consumer preferences over platform types are unaffected by the merger. Now suppose the merged entity can choose to either shut down one of the platforms or to keep both platforms under a joint ownership.

Let the monopoly profits of a single-platform monopolist and a two-platform monopolist be denoted $\Pi_1^m$ and $\Pi_2^m$ respectively. The merged entity will choose to shut down one platform if, and only if, $\Pi_1^m \geq \Pi_2^m$.

## 5.1 Single-platform monopoly

The profits of a single-platform monopolist that charges fixed fees to users that participate on the platform is of the form:

$$\Pi_1^m = (p_s - c_s)n_s + (p_b - c_b)n_b - f_1 \tag{49}$$

where $f_1$ is the monopolist's fixed costs.

In this setting there is only one platform in the market that buyers and sellers can choose to join. As before, the preferences of buyers and sellers are uniformly and continuously distributed over the interval $[0, 1]$ and the gross surplus of a buyer and a seller on the platform are again given by (31) and (32). Suppose that the monopolist chooses to shut down platform 1 so that the only platform in the market is located at 0. As before, the net utility of buyer located at $x \in [0, 1]$ is given by $U_b = w_b - x\tau_b$ from using the platform, and a seller located at $y \in [0, 1]$ obtains net utility $U_s = w_s - y\tau_s$ from joining the platform. A buyer located at $x \in [0, 1]$ chooses to join the platform if $U_b = w_b - x\tau_b \geq 0$ and a seller located at $y \in [0, 1]$ chooses to join the platform if $U_s = w_s - y\tau_s \geq 0$.

At initial pre-merger prices, the monopolist serves half of the market. When platform 1 is shut down, buyers and sellers located in $(\frac{1}{2}, 1]$ will join the monopolist's platform if it yields non-negative utility, and abstain from usage otherwise. I will consider the case when it is optimal for the monopolist to serve the whole market. The consumers located in $(\frac{1}{2}, 1]$ will join the monopolist's platform if the price is sufficiently low to compensate for the disutility arising from joining a less preferable platform. In addition, buyers must expect that a sufficiently large number of sellers choose to join the platform and vice versa. These expectations must be considered in the price setting of the monopolist for the analysis to be complete.

The buyer that is indifferent between joining the platform and abstaining participation is located at $\hat{x}$:

$$\hat{U}_b = w_b - \hat{x}\tau_b = 0 \Leftrightarrow \hat{x} = \frac{w_b}{\tau_b} \tag{50}$$

The indifferent seller is located at $\hat{y}$:

$$\hat{U}_s = w_s - \hat{y}\tau_s = 0 \Leftrightarrow \hat{y} = \frac{w_s}{\tau_s} \tag{51}$$

All the buyers located in $[0, \hat{x}]$ and all the sellers located in $[0, \hat{y}]$ choose to join the platform. The number of buyers and sellers that participate on the platform is thus given by:

$$n_b = \hat{x} = \frac{w_b}{\tau_b} \tag{52}$$

$$n_s = \hat{y} = \frac{w_s}{\tau_s} \tag{53}$$

By using (31) and (32) in the expressions above we obtain the number of buyers and sellers conditional on their expectations about the number of participants on the other side of the platform:

$$n_b = \frac{s_b + \alpha_b n_s - p_b}{\tau_b} \tag{54}$$

$$n_s = \frac{s_s + \alpha_s n_b - p_s}{\tau_s} \tag{55}$$

Solving the system of equations gives the fulfilled expectations equilibrium number of users on the platform:

$$n_b = \frac{\tau_s(s_b - p_b) + \alpha_b(s_s - p_s)}{\tau_b \tau_s - \alpha_b \alpha_s} \tag{56}$$

$$n_s = \frac{\tau_b(s_s - p_s) + \alpha_s(s_b - p_b)}{\tau_b \tau_s - \alpha_b \alpha_s} \tag{57}$$

The monopolist anticipates the behavior of the buyers and sellers and solves:

$$\max_{p_s, p_b} \Pi_1^m = \frac{1}{\tau_b \tau_s - \alpha_b \alpha_s} \Big[ (p_s - c_s)(\tau_b(s_s - p_s) + \alpha_s(s_b - p_b)) + (p_b - c_b)(\tau_s(s_b - p_b) + \alpha_b(s_s - p_s)) \Big] - f_1 \tag{58}$$

The derivation of the optimal prices and the conditions for when it is optimal for the monopolist to serve the entire market is tedious and is therefore relegated to appendix B.1. The profit maximizing monopoly prices coincide with full participation in equilibrium when:

$$2\tau_b(s_s - c_s) + (\alpha_b + \alpha_s)(s_b - c_b) \geq 4\tau_b\tau_s - (\alpha_b + \alpha_s)^2 \tag{59}$$

$$2\tau_s(s_b - c_b) + (\alpha_b + \alpha_s)(s_s - c_s) \geq 4\tau_b\tau_s - (\alpha_b + \alpha_s)^2 \tag{60}$$

It is only optimal for the monopolist to serve the entire market when the total benefits from joining a platform are sufficiently high for both buyers and sellers relative to their degree of platform differentiation.

The monopoly prices are then:

$$p_b = s_b + \alpha_b - \tau_b \tag{61}$$

$$p_s = s_s + \alpha_s - \tau_s \tag{62}$$

The monopoly profits are then given by:

$$\begin{aligned}\Pi_1^m &= (s_b + \alpha_b - \tau_b - c_b) + (s_s + \alpha_s - \tau_s - c_s - f_1) \\ &= s_b + s_s + \alpha_s + \alpha_b - \tau_b - \tau_s - c_b - c_s - f_1\end{aligned} \tag{63}$$

## 5.2 Two-platform monopoly

The profit of a two-platform monopolist that charges fixed fees is of the form:

$$\Pi_2^m = (p_s^0 - c_s)n_s^0 + (p_b^0 - c_b)n_b^0 + (p_s^1 - c_s)n_s^1 + (p_b^1 - c_b)n_b^1 - f_2 \tag{64}$$

where $f_2$ is the fixed costs of the two-platform monopolist.

It is assumed that $f_2 \geq f_1$. A substantial part of the fixed costs, in particular in the digital economy, arise from capital costs and costs related to research and development (R&D) and the maintenance of a platform (Duch-Brown 2017). It is logical to assume that the fixed costs of a monopolist operating two platforms are at least as high as the fixed costs of a single-platform monopolist.

As before, buyers and sellers are uniformly and continuously distributed over the interval $[0, 1]$ with respect to their preferences, and the platforms are located at the extreme points, 0 and 1 respectively. At initial, pre-merger, prices, the monopolist serves the entire market where each platform serves an equal share of consumers.

I will again consider the case where it is profit maximizing for the monopolist to opt for a covered market. Since marginal costs are unaffected by the merger there is no reason to assume that the monopolist will charge asymmetric prices. Hence, $p_b^1 = p_b^2 = p_b$ and $p_s^1 = p_s^2 = p_s$. Recall that under full market coverage, symmetric prices imply that the consistent expectations equilibrium network sizes always equals $\frac{1}{2}$ for both sides of the market. That is, platforms serve an equal share of buyers and sellers in equilibrium. The best the monopolist can do is then to set the highest prices that the indifferent buyer and seller can accept, that is, the price that gives the indifferent users zero utility.

By the full market participation constraints, (FMPCB2) and (FMPCS2), it then follows that the utility of the indifferent buyer and seller is given by $\hat{U}_b = \frac{1}{2}[\alpha_b - 2p_b - \tau_b + 2s_b]$ and $\hat{U}_s = \frac{1}{2}[\alpha_s - 2p_s - \tau_s + 2s_s]$ respectively. The two-platform monopolist will set the fees according to:

$$\frac{1}{2}[\alpha_b - 2p_b - \tau_b + 2s_b] = 0$$
$$\iff p_b = \frac{1}{2}[\alpha_b - \tau_b + 2s_b] \tag{65}$$
$$\frac{1}{2}[\alpha_s - 2p_s - \tau_s + 2s_s] = 0$$
$$\iff p_s = \frac{1}{2}[\alpha_s - \tau_s + 2s_s] \tag{66}$$

The monopoly profit of a two-platform monopolist is then:

$$\begin{aligned}\Pi_2^m = (p_s - c_s)\frac{1}{2} + (p_b - c_b)\frac{1}{2} + (p_s - c_s)\frac{1}{2} + (p_b - c_b)\frac{1}{2} - f_2 \\ = (p_s - c_s) + (p_b - c_b) - f_2 \\ = (\frac{1}{2}[\alpha_s - \tau_s + 2s_s] - c_s) + (\frac{1}{2}[\alpha_b - \tau_b + 2s_b] - c_b) - f_2 \\ = \frac{1}{2}(\alpha_s + \alpha_b - \tau_s - \tau_b) + s_s + s_b - c_s - c_b - f_2\end{aligned} \tag{67}$$

# 6 The decision of the merged entity

The merged entity is better off shutting down a platform and keeping only one, if and only if:

$$
\begin{aligned}
\Pi_1^m = \alpha_s + \alpha_b - \tau_b - \tau_s + s_b + s_s - c_b - c_s - f_1 \geq \\
\frac{1}{2}(\alpha_s + \alpha_b - \tau_s - \tau_b) + s_s + s_b - c_s - c_b - f_2 = \Pi_2^m \\
\iff \frac{1}{2}(\alpha_s + \alpha_b - \tau_s - \tau_b) + f_2 - f_1 \geq 0 \\
\iff \alpha_s + \alpha_b - \tau_s - \tau_b + 2(f_2 - f_1) \geq 0 \quad (68)
\end{aligned}
$$

Since $f_2 \geq f_1$ the last term on the left hand side of the inequality sign is always non-negative. In particular, the following holds.

**Proposition 3** *Let $\Delta f = f_2 - f_1$ denote the difference in fixed costs. The merged entity will choose to shut down one platform if, and only if:*

$$\Delta f \geq \frac{1}{2}(\tau_b + \tau_s - \alpha_b - \alpha_s) \quad (69)$$

*and keep both platforms under a joint ownership otherwise.*

*Also, the merged entity will choose to keep both platforms for any parameter values that satisfy the conditions in the model if:*

$$\Delta f \leq \frac{1}{2}(\sqrt{\tau_b} - \sqrt{\tau_s})^2 \quad (70)$$

*Proof.* The expression in (69) follows from the definition of $\Delta f$ and the inequality in (68).

By condition (EC) it follows that, for any $\tau_b, \tau_s, \alpha_b, \alpha_s > 0$:

$$\tau_b + \tau_s - \alpha_b - \alpha_s > \tau_b + \tau_s - 2\sqrt{\tau_b \tau_s} = (\sqrt{\tau_b} - \sqrt{\tau_s})^2$$

This implies that it is necessary that the following condition is satisfied for (69) to hold:

$$\Delta f > \frac{1}{2}(\sqrt{\tau_b} - \sqrt{\tau_s})^2 \quad (71)$$

From this it follows that (69) can never hold if:

$$\Delta f \leq \frac{1}{2}(\sqrt{\tau_b} - \sqrt{\tau_s})^2 \tag{72}$$

■

First, note that in absence of fixed cost ($\Delta f = 0$), condition (70) will always hold. Hence, in case there are no fixed costs, the merged entity always choose to keep both platforms which coincides with the conclusion in Tan & Zhou (2019).

Second, we observe that a merged entity is more likely to keep both platforms if the difference in fixed costs is small. If a merger is associated with large savings in fixed costs, the merged entity is more likely to choose to operate both platforms under a joint ownership. However, by condition (69) it can be noted that if the indirect network effects are strong in relation to the buyers' and sellers' degree of platform differentiation, then it may be more profitable for the merged entity to shut down one platform even for small differences in fixed costs.

From condition (70) it can be noticed that in case there are large asymmetries between the buyers' and sellers' degree of platform differentiation, the merged entity is better off keeping two platforms even when the cost of maintaining a second platform is large.

# 7 Merger analysis

I will analyze how the merger affects the consumer surplus. As mentioned in the introduction this is an appropriate focus since antitrust authorities tend to use the consumer surplus as the relevant measure of welfare in merger investigations.

In order to examine the effects of a merger on the consumer surplus it is necessary to derive the consumer surplus in the pre- and post-merger state. The consumer surplus is the sum of the total buyer and seller surpluses:

$$CS = BS + SS \tag{73}$$

where $BS$ and $SS$ denote the buyers' and sellers' respective surpluses. The total surplus of a group is given by the sum of each group-member's net surplus from joining a platform.

Recall that buyers and sellers are continuously and uniformly distributed over the unit interval. Platforms are located at the endpoints of the interval in the case there are two platforms in the market. Recall also that the share of buyers and sellers that joins platform 0 is given by $n_b^0$ and $n_s^0$ respectively, and the proportion of buyers and sellers that joins platform 1 is given by $n_b^1 = 1 - n_b^0$ and $n_s^1 = 1 - n_s^0$ respectively. The net surplus of a buyer located at $x \in [0, n_b^0]$ is given by $U_b^0 = w_b^0 - x\tau_b$, and the net surplus of a buyer located at $x \in (n_b^0, 1]$ is given by $U_b^1 = w_b^1 - (1-x)\tau_b$.

The total buyer surplus is given by the sum of the net surpluses of all individual buyers that use platform 0 and 1. Hence, the total buyer surplus when there are two platforms in the market is given by:

$$
\begin{aligned}
BS_2 &= \int_0^{n_b^i} U_b^i(x)dx + \int_{n_b^i}^1 U_b^{-i}(x)dx \\
&= \int_0^{n_b^i} (w_b^i - x\tau_b)dx + \int_{n_b^i}^1 (w_b^{-i} - (1-x)\tau_b)dx \\
&= \int_0^{n_b^i} (s_b + n_s^i\alpha_b - p_b^i - x\tau_b)dx + \int_{n_b^i}^1 (s_b + n_s^{-i}\alpha_b - p_b^{-i} - \tau_b + x\tau_b)dx \quad (74)
\end{aligned}
$$

Analoguously, the total seller surplus is given by:

$$
\begin{aligned}
SS_2 &= \int_0^{n_s^i} U_s^i(y)dy + \int_{n_s^i}^1 U_s^{-i}(y)dy \\
&= \int_0^{n_s^i} (w_s^i - y\tau_s)dy + \int_{n_s^i}^1 (w_s^{-i} - (1-y)\tau_s)dy \\
&= \int_0^{n_s^i} (s_s + n_b^i\alpha_s - p_s^i - y\tau_s)dy + \int_{n_s^i}^1 (s_s + n_b^{-i}\alpha_s - p_s^{-i} - \tau_s + y\tau_s)dy \quad (75)
\end{aligned}
$$

In case there is only one platform in the market that is located at one of the endpoints of the unit interval, the total surplus of buyers and sellers is respectively given by:

$$BS_1 = \int_0^{n_b^i} U_b^i(x)dx = \int_0^{n_b^i} (w_b^i - x\tau_b)dx = \int_0^{n_b^i} (s_b + n_s^i\alpha_b - p_b^i - x\tau_b)dx \quad (76)$$

and

$$SS_1 = \int_0^{n_s^i} U_s^i(y)dy = \int_0^{n_s^i} (w_s^i - y\tau_s)dy = \int_0^{n_s^i} (s_s + n_b^i\alpha_s - p_s^i - y\tau_s)dy \quad (77)$$

## 7.1 Pre-merger surplus

To obtain the pre-merger buyer and seller surpluses we insert the duopoly prices, (24) and (25), and corresponding network sizes, (26) and (27), in (74) and (75):

$$\begin{aligned} BS^{pre} &= \int_0^{\frac{1}{2}} (s_b + \frac{1}{2}\alpha_b - (c_b + \tau_b - \alpha_s) - x\tau_b)dx \\ &\quad + \int_{\frac{1}{2}}^{1} (s_b + \frac{1}{2}\alpha_b - (c_b + \tau_b - \alpha_s) - \tau_b + x\tau_b)dx \\ &= \int_0^{\frac{1}{2}} (s_b + \frac{1}{2}\alpha_b + \alpha_s - c_b - \tau_b - x\tau_b)dx + \int_{\frac{1}{2}}^{1} (s_b + \frac{1}{2}\alpha_b + \alpha_s - c_b - 2\tau_b + x\tau_b)dx \\ &= s_b + \frac{1}{2}\alpha_b + \alpha_s - c_b - \frac{5}{4}\tau_b \end{aligned} \tag{78}$$

$$\begin{aligned} SS^{pre} &= \int_0^{\frac{1}{2}} (s_s + \frac{1}{2}\alpha_s - (c_s + \tau_s - \alpha_b) - y\tau_s)dy \\ &\quad + \int_{\frac{1}{2}}^{1} (s_s + \frac{1}{2}\alpha_s - (c_s + \tau_s - \alpha_b) - \tau_s + y\tau_s)dy \\ &= \int_0^{\frac{1}{2}} (s_s + \frac{1}{2}\alpha_s + \alpha_b - c_s - \tau_s - y\tau_s)dy + \int_{\frac{1}{2}}^{1} (s_s + \frac{1}{2}\alpha_s + \alpha_b - c_s - 2\tau_s + y\tau_s)dy \\ &= s_s + \frac{1}{2}\alpha_s + \alpha_b - c_s - \frac{5}{4}\tau_s \end{aligned} \tag{79}$$

The pre-merger consumer surplus is given by the sum of the buyers' and sellers' pre-merger surpluses:

$$CS^{pre} = BS^{pre} + SS^{pre} = s_b + s_s + \frac{3}{2}(\alpha_b + \alpha_s) - (c_b + c_s) - \frac{5}{4}(\tau_b + \tau_s) \tag{80}$$

## 7.2 Post-merger surplus

### 7.2.1 Two-platform monopoly

To obtain the buyer and seller surpluses under a two-platform monopoly we insert the monopoly prices given in (65) and (66) and the network sizes, $n_b^1 = n_s^1 = n_b^2 = n_s^2 = \frac{1}{2}$, in (74) and (75):

$$BS_2^{post} = \int_0^{\frac{1}{2}} (s_b + \frac{1}{2}\alpha_b - \frac{1}{2}(\alpha_b - \tau_b + 2s_b) - x\tau_b)dx$$
$$+ \int_{\frac{1}{2}}^{1} (s_b + \frac{1}{2}\alpha_b - \frac{1}{2}(\alpha_b - \tau_b + 2s_b) - \tau_b + x\tau_b)dx$$
$$= \int_0^{\frac{1}{2}} (\frac{1}{2}\tau_b - x\tau_b)dx + \int_{\frac{1}{2}}^{1} (-\frac{1}{2}\tau_b + x\tau_b)dx = \frac{1}{4}\tau_b \quad (81)$$

$$SS_2^{post} = \int_0^{\frac{1}{2}} (s_s + \frac{1}{2}\alpha_s - \frac{1}{2}(\alpha_s - \tau_s + 2s_s) - y\tau_b)dy$$
$$+ \int_{\frac{1}{2}}^{1} (s_s + \frac{1}{2}\alpha_s - \frac{1}{2}(\alpha_s - \tau_s + 2s_s) - \tau_s + y\tau_s)dy$$
$$= \int_0^{\frac{1}{2}} (\frac{1}{2}\tau_s - y\tau_s)dy + \int_{\frac{1}{2}}^{1} (-\frac{1}{2}\tau_s + y\tau_s)dy = \frac{1}{4}\tau_s \quad (82)$$

The post-merger consumer surplus is given by the sum of the buyers' and sellers' post-merger surpluses:

$$CS_2^{post} = BS_2^{post} + SS_2^{post} = \frac{1}{4}(\tau_b + \tau_s) \quad (83)$$

### 7.2.2 Single-platform monopoly

To obtain the buyer and seller surpluses under a single-platform monopoly that covers the entire market we insert the monopoly prices given in (61) and (62) and the network sizes, $n_b = n_s = 1$, in (76) and (77):

$$BS_1^{post} = \int_0^1 (s_b + \alpha_b - (s_b - \tau_b + \alpha_b) - x\tau_b)dx$$
$$= \int_0^1 (\tau_b(1 - x))dx = \frac{1}{2}\tau_b \quad (84)$$

$$SS_1^{post} = \int_0^1 (s_s + \alpha_s - (s_s - \tau_s + \alpha_s) - y\tau_s)dy$$
$$= \int_0^1 (\tau_s(1 - y))dy = \frac{1}{2}\tau_s \quad (85)$$

The post-merger consumer surplus is given by the sum of the buyers' and sellers' post-merger surpluses:

$$CS_1^{post} = BS_1^{post} + SS_1^{post} = \frac{1}{2}(\tau_b + \tau_s) \qquad (86)$$

It is now uncomplicated to prove the following.

**Proposition 4** *The consumer surplus is always higher under a single-platform monopoly that serves the entire market than under a two-platform monopoly.*

*Proof.* From (83) and (86) it is easily noted that the consumer surplus under a single-platform monopoly that covers the entire market is twice as high as the consumer surplus under a two-platform monopoly that covers the entire market:

$$CS_1^{post} = \frac{1}{2}(\tau_b + \tau_s) = \frac{2}{4}(\tau_b + \tau_s) = 2 \cdot CS_2^{post}$$

Obviously, the consumer surplus is higher under a two-platform monopoly that serves the entire market than under a two-platform monopoly with partial market coverage. Hence, proposition 4 follows. ■

## 7.3 Comparison of pre- and post-merger surpluses

### 7.3.1 Two-platform monopoly

It is also straightforward to show the following.

**Proposition 5** *The consumer surplus can never increase as a result of a merger when the merged entity chooses to keep two platforms.*

*Proof.* Recall that parameter values must satisfy the full market participation conditions (FMPCB2*) and (FMPCS2*), repeated here for the convenience of the reader:

$$\alpha_b + 2\alpha_s - 3\tau_b - 2c_b + 2s_b \geq 0$$
$$\alpha_s + 2\alpha_b - 3\tau_s - 2c_s + 2s_s \geq 0$$

It follows that their sum must be non-negative:

$$\begin{aligned}
&2(s_b + s_s - c_b - c_s) + 3(\alpha_b + \alpha_s - \tau_b - \tau_s) \geq 0 \\
&\iff s_b + s_s - c_b - c_s + \frac{3}{2}(\alpha_b + \alpha_s) - \frac{3}{2}(\tau_b + \tau_s) \geq 0 \\
&\iff s_b + s_s - c_b - c_s + \frac{3}{2}(\alpha_b + \alpha_s) - \frac{5}{4}(\tau_b + \tau_s) - \frac{1}{4}(\tau_b + \tau_s) \geq 0 \\
&\iff CS^{pre} - CS_2^{post} \geq 0 \\
&\iff CS^{pre} \geq CS_2^{post} \qquad (87)
\end{aligned}$$

where the second latest equivalence follows from (80) and (83). The pre-merger consumer surplus is at least as high as the post-merger surplus under a two-platform monopoly with full market coverage. Hence, the consumer surplus cannot increase after a merger when the merged entity's optimal choice is to keep both platforms. Since the consumer surplus cannot increase as a result of a merger into a two-platform monopoly under the assumption that it is optimal for the merged entity to serve the whole market, it should be evident that it will neither increase in the case that the price setting results in a partially covered market. ■

### 7.3.2 Single-platform monopoly

The consumer surplus increases as a result of a merger from a duopoly to a single-platform monopoly if:

$$CS_1^{post} > CS^{pre} \qquad (88)$$

Inserting the expression for the pre- and post-merger surplus, (80) and (86) yields:

$$\begin{aligned}
&\frac{1}{2}(\tau_b + \tau_s) > s_b + s_s + \frac{3}{2}(\alpha_b + \alpha_s) - (c_b + c_s) - \frac{5}{4}(\tau_b + \tau_s) \\
&\iff s_b + s_s < c_b + c_s + \frac{7}{4}(\tau_b + \tau_s) - \frac{3}{2}(\alpha_b + \alpha_s) \qquad (89)
\end{aligned}$$

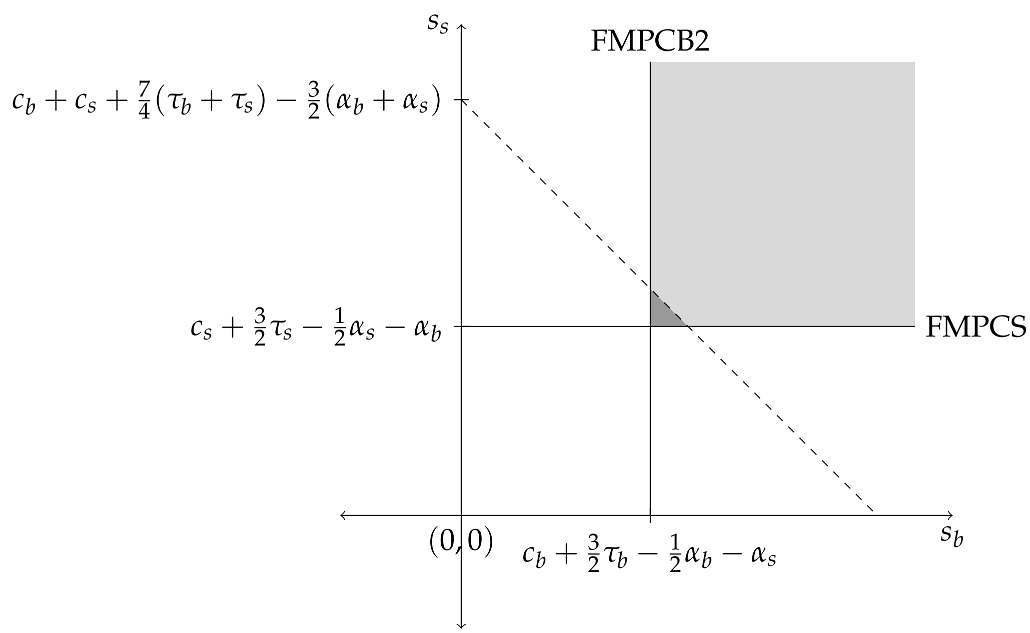
where $\frac{7}{4}(\tau_b + \tau_s) - \frac{3}{2}(\alpha_b + \alpha_s) > 0$ by the Equilibrium Condition (EC). The consumer surplus increases as a result of a merger into a single-platform monopoly if the stand-alone benefits are sufficiently low. However, recall that the conditions for a covered market implies a lower bound on the stand-alone benefits. By the full market participation conditions we have that:

$$s_b \geq c_b + \frac{3}{2}\tau_b - \frac{1}{2}\alpha_b - \alpha_s \tag{90}$$

$$s_s \geq c_s + \frac{3}{2}\tau_s - \frac{1}{2}\alpha_s - \alpha_b \tag{91}$$

It is the case that the upper bounds on the stand-alone benefits given by (89), are compatible with the lower bounds on the stand-alone benefits given by the conditions for a covered market.[14] Hence, it is possible that the consumer surplus increases as a result of a merger into a single-platform monopoly that covers the entire market. For fixed values of the other parameters it can be illustrated in a $(s_b, s_s)$-plane:

Figure 4




The shaded area represent the set of combinations of $(s_b, s_s)$ for which the full market participation conditions are satisfied for buyers and sellers. The darker shaded triangular shaped area is the set of combinations of $(s_b, s_s)$ for which the conditions

[14] There exist combinations of parameter values that satisfies (89),(FMPCB2*),(FMPCS2*) and (EC). The solution set is of little practical relevance and is therefore not included in this thesis. The solution in the case with perfect symmetry is displayed in appendix (B.2) for the interested reader.

for a covered market are fulfilled and for which the consumer surplus increases as a result of a merger into a single-platform monopoly. Note that the opposing restrictions on the stand-alone benefits implies that the a pair $(s_b, s_s)$ must be sufficiently close to the full market participation constraints of buyers and sellers in order to satisfy both the conditions for a covered market and the condition for a post-merger increase in consumer surplus.

The result can be summarized as follows.

**Proposition 6** *The consumer surplus increases as a result of a merger into a single-platform monopoly that covers the entire market if the following conditions are all satisfied:*

$$\begin{aligned} s_b + s_s < c_b + c_s + \frac{7}{4}(\tau_b + \tau_s) - \frac{3}{2}(\alpha_b + \alpha_s) \\ s_b \geq c_b + \frac{3}{2}\tau_b - \frac{1}{2}\alpha_b - \alpha_s \\ s_s \geq c_s + \frac{3}{2}\tau_s - \frac{1}{2}\alpha_s - \alpha_b \end{aligned}$$

Consider for a moment the first inequality in proposition 6 and rewrite it as:

$$s_b + s_s + \frac{3}{2}(\alpha_b + \alpha_s) < c_b + c_s + \frac{7}{4}(\tau_b + \tau_s)$$

The stand-alone benefits and the benefits from the interaction must be sufficiently low in relation to the marginal costs and the degree of product differentiation for the consumer surplus to increase as a result of a merger into a single-platform monopoly. This may at first seem counterintuitive since a high degree of product differentiation implies that consumers with the highest preferences for the platform type that is shut down also have considerably lower preferences for the platform type that remains on the market. It can be explained by the assumption that it is optimal for the monopolist to serve the whole market. Recall that in the case that the optimal monopoly prices coincide with a covered market, the single-platform monopolist set their prices according to:

$$p_k = s_k + \alpha_k - \tau_k \qquad\qquad k \in \{b, s\}$$

The prices increase in the stand-alone benefits and the indirect network effects, and decrease in the degree of platform differentiation. The single-platform monopolist sets the prices to each group equal to the total benefit of joining its platform less the degree of perceived platform differentiation. Since the monopolist cannot price discriminate, it subsidizes every consumer by $\tau_k$, which is the minimum compensation so that the consumer with the lowest preferences for the monopolist's platform type is exactly compensated for the disutility of choosing his or her least preferred platform type. The net utility of an agent in group $k$, that is located at $x \in [0, 1]$ is:

$$w_k = \tau_k - x\tau_k \tag{92}$$

It follows that the net utility of every agent located at $x \in [0, 1)$ increases in the degree of platform differentiation:

$$\frac{dw_k}{d\tau_k} = 1 - x > 0 \qquad \text{for any } x \in [0, 1) \tag{93}$$

and by that, the consumer surplus.

# 8 Discussion and Conclusion

In this thesis, I have analyzed a horizontal merger from a duopoly to a monopoly in a two-sided market, with digital platform markets in mind. In the first part of the thesis, I derived the pre-merger equilibrium by following an analysis of competition in two-sided markets proposed by Armstrong (2006). I found that the model where the consumer groups only value a platform by its ability to enable their interaction is not useful for a merger analysis. Given the restrictions on the parameters for a unique market sharing equilibrium, the platforms don't share market areas in equilibrium in these settings. I then established that stand-alone benefits for at least one side of the market are necessary to ensure that the conditions for an overlapping market area are compatible with the necessary conditions for a unique market sharing equilibrium. This precluded an analysis of mergers in markets that lack such benefits such as e-commerce marketplaces. I thus considered the effects of a horizontal merger in digital platform markets where the assumption fits, as for example the digital newspapers market and the market for online advertising platforms and online intermediation.

In the second part of the thesis, I derived the post-merger equilibrium under two possible scenarios: a merger into a joint ownership of the two platforms and the case where the merged entity shuts down one platform. I found that the merged entity has incentives to shut down one platform if the fixed costs from maintaining a platform are sufficiently large. If a merger is related to large savings in fixed costs the merged entity is more likely to keep both platforms. In the absence of fixed costs, the merged entity has no incentives to shut down one platform and operate the remaining one, which goes in line with the conclusions in Tan & Zhou (2019). The finding that the decision of the merged entity is dependent on fixed costs is relevant since high fixed costs prevail in many two-sided industries, particularly in digital two-sided markets.

An important finding is that the consumer surplus always decreases as a result of a merger into a joint ownership of the platforms. It can then be concluded that mergers in these types of markets should raise concerns when there are high savings in fixed costs related to the merger. Also, by the result in proposition 3, antitrust authorities should be particularly vigilant if the market is characterized by two asymmetric sides in terms of how the two consumer groups perceive the degree of platform differentiation. In such environments, it is more profitable for the merged entity to keep two platforms even absent large savings in fixed costs.

My conclusions concerning the effects on consumer welfare from a merger into a joint ownership of the two platforms contrast with previous studies that find that such mergers may be welfare enhancing (e.g. Chandra & Collard-Wexler (2008), Leonello (2011), Cosnita-Langlais et al. (2016)). My results are conditioned on the assumption that the merged entity does not make the platforms interoperable and that there are no efficiency gains. I maintain that efficiency gains are presumably of little relevance for a merger analysis in the types of markets I consider. In digital platform markets marginal costs are close to zero to begin with. However, it may be relevant to consider the scenario where the merged entity makes the platforms interoperable. In the model, interoperability could be represented by a decrease in consumers' disutility cost from choosing a less preferred platform type. As is also highlighted in Correia-da-Silva et al. (2019), the merged entity would then have to find an optimal balance between the benefits of network effects generated by interoperability and the benefits of platforms' differentiation.

I also found that the consumer surplus can increase as a result of a merger into a single-platform monopoly that covers the entire market if the total benefits from joining the platform are sufficiently small in relation to consumers' perceived degree of platform differentiation. While this may at first seem counter-intuitive, it is explained by the price setting of a single-platform monopolist that serves the entire market. Under complete market coverage, the monopoly platform compensates each consumer for their disutility cost from choosing a less preferred platform type. At the same time, the monopolist's profit maximizing prices only coincide with full market coverage when the consumers' perceived degree of platform differentiation is small in relation to their total benefits from joining the platform.[15] These conflicting conditions should be investigated more thoroughly to identify when (or if) they coincide. By the analysis in this thesis we may just conclude that it is ambiguous whether a merger into a single-platform monopoly will increase consumer surplus.

Similar to many previous studies of horizontal mergers in two-sided markets I have assumed that agents only patronize one platform. This is arguably a specific assumption, in particular in the case of digital two-sided markets. The case where consumers have the possibility to join more than one platform should be addressed in the research on mergers in two-sided markets to fully comprehend the effects of increased concentration in digital platform markets.

Some concluding remarks should also be made regarding the first part of this thesis. First of all, the conclusion that the model proposed by Armstrong (2006) is incomplete absent stand-alone benefits suggests that adaptations of this model should be completed with such parameters, as for example in Belleflamme & Peitz (2010).

Second, a model with stand-alone benefits have limited areas of applications. The prevalence of stand-alone benefits implies that users obtain some utility from using the platform irrespective of the number of participants on the other side of the market. This is motivated if the platforms provide some content or service in addition to enabling the interaction between the groups of consumers.

However, many digital two-sided markets lack such features and in many cases

[15] Recall that in section 5.1 it was established that it is only optimal for a single-platform monopolist to serve the entire market when total benefits from joining a platform are sufficiently high for buyers and sellers.

agents only value a platform by its ability to enable the interaction with other groups of agents. What we can conclude is that when agents patronize only one platform, horizontal platform differentiation may be insufficient to enable the co-existence of more platforms in equilibrium in such environments. Undoubtedly, there is a need for more research on these market types, especially due to the high prevalence of digital markets with the only function to enable the interaction between two or more groups of consumers.

# 9 Bibliography


Armstrong, M. (2006), 'Competition in two-sided markets', RAND Journal of Economics **37**, 668–691.

Belleflamme, P. & Peitz, M. (2010), Industrial Organization: Markets and Strategies, Cambridge University Press.
**URL:** *https://books.google.se/books?id=nmUcldoQkagC*

Caillaud, B. & Jullien, B. (2003), 'Chicken & Egg: Competition among Intermediation Service Providers', RAND Journal of Economics **34**(2), 309–328.
**URL:** *https://ideas.repec.org/a/rje/randje/v34y2003i2p309-28.html*

Chandra, A. & Collard-Wexler, A. (2008), Mergers in Two-Sided Markets: An Application to the Canadian Newspaper Industry, MPRA Paper 7954, University Library of Munich, Germany.
**URL:** *https://ideas.repec.org/p/pra/mprapa/7954.html*

Correia-da-Silva, J., Jullien, B., Lefouili, Y. & Pinho, J. (2019), 'Horizontal mergers between multisided platforms: Insights from Cournot competition', Journal of Economics & Management Strategy **28**(1), 109–124.

Cosnita-Langlais, A., Baranes, E. & Cortade, T. (2016), Merger control on two-sided markets: is there need for an efficiency defense?, Technical report.

Damiano, E. & Li, H. (2008), 'Competing Matchmaking', Journal of the European Economic Association **6**(4), 789–818.
**URL:** *https://doi.org/10.1162/JEEA.2008.6.4.789*

Duch-Brown, N. (2017), The competitive landscape of online platforms, Working paper.
**URL:** *https://ec.europa.eu/jrc/sites/jrcsh/files/jrc106299.pdf*

Evans, D. & Schmalensee, R. (2017), Multi-sided Platforms, pp. 1–9.

Filistrucchi, L., Klein, T. J. & Michielsen, T. O. (2012), 'Assessing unilateral merger effects in a two-sided market: an application to the Dutch daily newspaper market', Journal of Competition Law  Economics **8**(2), 297–329.
**URL:** *https://doi.org/10.1093/joclec/nhs012*

Jeziorski, P. (2014), 'Effects of mergers in two-sided markets: The us radio industry', American Economic Journal: Microeconomics **6**(4), 35–73.
**URL:** *https://www.aeaweb.org/articles?id=10.1257/mic.6.4.35*

Leonello, A. (2011), 'Three essays in competition and banking', Doctoral dissertation, European University Institute pp. 83–106.
**URL:** *https://cadmus.eui.eu/bitstream/handle/1814/18394/2011*$_l eonello.pdf?sequence = 1 is Allowed = y$

Rochet, J.-C. & Tirole, J. (2003), Platform Competition in Two-Sided Markets, Technical report.

Rochet, J.-C. & Tirole, J. (2006), 'Two-sided markets: a progress report', The RAND Journal of Economics **37**(3), 645–667.
**URL:** *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1756-2171.2006.tb00036.x*

Rysman, M. (2009), 'The Economics of Two-Sided Markets', Journal of Economic Perspectives **23**(3), 125–143.
**URL:** *https://ideas.repec.org/a/aea/jecper/v23y2009i3p125-43.html*

Tan, G. & Zhou, J. (2019), 'Price competition in multi-sided markets', SSRN Electronic Journal .
**URL:** *https://ssrn.com/abstract=3029134*

Van Cayseele, P. & Vanormelingen, S. (2019), 'Merger analysis in two-sided markets: The belgian newspaper industry', Review of Industrial Organization **54**(3), 509–541.

Weyl, E. G. (2010), 'A price theory of multi-sided platforms', American Economic Review **100**(4), 1642–72.
**URL:** *https://www.aeaweb.org/articles?id=10.1257/aer.100.4.1642*

# Appendices

## A

### A.1 Proof for Proposition 1

The necessary and sufficient condition for a market sharing interior solution to exist is:

$$4\tau_b\tau_s > (\alpha_s + \alpha_b)^2 \tag{A.1.1}$$

The market is covered in equilibrium if the following two conditions hold:

$$\alpha_b + 2\alpha_s - 3\tau_b - 2c_b \geq 0 \tag{A.1.2}$$

$$\alpha_s + 2\alpha_b - 3\tau_s - 2c_s \geq 0 \tag{A.1.3}$$

Where $\alpha_b, \alpha_s, \tau_b, \tau_s > 0$ and $c_b, c_s \geq 0$. Conditions (A.1.2) and (A.1.3) can be rearranged as follows:

$$\alpha_b + 2\alpha_s \geq 3\tau_b + 2c_b \geq 3\tau_b \tag{A.1.4}$$

$$\alpha_s + 2\alpha_b \geq 3\tau_s + 2c_s \geq 3\tau_s \tag{A.1.5}$$

Given that $c_b, c_s \geq 0$ the rightmost inequality in (A.1.4) and (A.1.5) will always hold. To summarize, it must hold that:

$$4\tau_b\tau_s > (\alpha_s + \alpha_b)^2 \tag{A.1.6}$$

$$\alpha_b + 2\alpha_s \geq 3\tau_b \tag{A.1.7}$$

$$\alpha_s + 2\alpha_b \geq 3\tau_s \tag{A.1.8}$$

Let $\mathcal{S}$ denote the set of possible solutions to the system of inequalities (A.1.1), (A.1.2) and (A.1.3), and let $\mathcal{P}$ denote the set of possible solutions to the system of inequalities (A.1.6), (A.1.7) and (A.1.8). It should be evident that $\mathcal{S} \subset \mathcal{P}$. If it can be shown that $\mathcal{P} = \emptyset$, it immediately follows that $\mathcal{S} = \emptyset$ and by this it can be concluded that (A.1.1), (A.1.2) and (A.1.3) are contradictory.

Solving for $\tau_b$ and $\tau_s$ in (A.1.7) and (A.1.8) yields:

$$\tau_b \leq \frac{1}{3}\alpha_b + \frac{2}{3}\alpha_s$$
$$\tau_s \leq \frac{1}{3}\alpha_s + \frac{2}{3}\alpha_b$$

It then follows that:

$$\begin{aligned} 4\tau_b\tau_s &\leq 4(\frac{1}{3}\alpha_b + \frac{2}{3}\alpha_s)(\frac{1}{3}\alpha_s + \frac{2}{3}\alpha_b) \\ &= \frac{8}{9}(\alpha_b^2 + \frac{5}{2}\alpha_b\alpha_s + \alpha_s^2) \end{aligned}$$

From condition (A.1.1) we had that $4\tau_b\tau_s > (\alpha_s + \alpha_b)^2 = \alpha_b^2 + 2\alpha_b\alpha_s + \alpha^2$. Hence, we get that it is necessary that the following hold for the conditions to be satisfied:

$$\alpha_b^2 + 2\alpha_b\alpha_s + \alpha^2 < 4\tau_b\tau_s \leq \frac{8}{9}(\alpha_b^2 + \frac{5}{2}\alpha_b\alpha_s + \alpha_s^2)$$

It is therefore necessary that the following hold:

$$\begin{aligned} &\alpha_b^2 + 2\alpha_b\alpha_s + \alpha_s^2 < \frac{8}{9}(\alpha_b^2 + \frac{5}{2}\alpha_b\alpha_s + \alpha_s^2) \\ &\iff \alpha_b^2 + 2\alpha_b\alpha_s + \alpha_s^2 < \frac{8}{9}(\alpha_b^2 + 2\alpha_b\alpha_s + \alpha_s^2) + \frac{4}{9}\alpha_b\alpha_s \\ &\iff \frac{1}{9}(\alpha_b^2 + 2\alpha_b\alpha_s + \alpha_s^2) - \frac{4}{9}\alpha_b\alpha_s < 0 \\ &\iff \alpha_b^2 + 2\alpha_b\alpha_s + \alpha_s^2 - 4\alpha_b\alpha_s < 0 \\ &\iff \alpha_b^2 - 2\alpha_b\alpha_s + \alpha_s^2 = (\alpha_b - \alpha_s)^2 < 0 \end{aligned}$$

Now it is easily noticed that this is a contradiction. Since the necessary conditions for a solution the system of inequalities (A.1.6), (A.1.7) and (A.1.8) are contradictory, it is straightforward to conclude that there exist no solution to this system of inequalities. Thereby $\mathcal{P} = \emptyset$ and since $\mathcal{P} \subset \mathcal{S}$ it follows that $\mathcal{S} = \emptyset$ and that (A.1.1), (A.1.2) and (A.1.3) are contradictory.

## A.2 Proof for Proposition 2

The necessary and sufficient condition for a market sharing interior solution to exist is:

$$4\tau_b\tau_s > (\alpha_s + \alpha_b)^2 \tag{A.2.1}$$

The market is covered in equilibrium if the following two conditions hold:

$$\alpha_b + 2\alpha_s - 3\tau_b + 2(s_b - c_b) \geq 0 \tag{A.2.2}$$

$$\alpha_s + 2\alpha_b - 3\tau_s + 2(s_s - c_s) \geq 0 \tag{A.2.3}$$

Where $\alpha_b, \alpha_s, \tau_b, \tau_s > 0$ and $s_b, s_s, c_b, c_s \geq 0$.

Condition (A.2.1) is equivalent to:

$$2\sqrt{\tau_b \tau_s} > \alpha_b + \alpha_s$$

since all parameters are positive.

Now let $s_s = 0$. Then the system of inequalities (expressed in terms of $\alpha_s$) reduces to:

$$\alpha_s < 2\sqrt{\tau_b \tau_s} - \alpha_b \tag{A.2.4}$$

$$\alpha_s \geq \frac{3}{2}\tau_b - (s_b - c_b) - \frac{1}{2}\alpha_b \tag{A.2.5}$$

$$\alpha_s \geq 3\tau_s + 2c_s - 2\alpha_b \tag{A.2.6}$$

Note that (A.2.4), (A.2.5) and (A.2.6) are stricter restrictions on the parameters than (A.2.1), (A.2.2) and (A.2.3). If parameter values can satisfy the conditions when only one side of the market obtains positive stand-alone benefits, it follows that there also exists parameter values that satisfy the conditions when both side of the market obtain positive stand-alone benefits. It therefore suffices to examine the case where only one side of the market obtains stand-alone benefits.

Now, I find the restrictions on the parameters such that conditions (A.2.5) and (A.2.6) are exactly satisfied by setting the conditions to equality and finding their point of intersection, $P = (\alpha_b^P, \alpha_s^P)$.

$$\alpha_s^P = \frac{3}{2}\tau_b - (s_b - c_b) - \frac{1}{2}\alpha_b^P \tag{A.2.7}$$

$$\alpha_s^P = 3\tau_s + 2c_s - 2\alpha_b^P \tag{A.2.8}$$

$$\implies \frac{3}{2}\tau_b - (s_b - c_b) - \frac{1}{2}\alpha_b^P = 3\tau_s + 2c_s - 2\alpha_b^P$$
$$\iff \alpha_b^P = 2\tau_s - \tau_b + \frac{4}{3}c_s + \frac{2}{3}(s_b - c_b) \tag{A.2.9}$$

$$\implies \alpha_s^P = 3\tau_s + 2c_s - 2(2\tau_s - \tau_b + \frac{4}{3}c_s + \frac{2}{3}(s_b - c_b))$$
$$= 2\tau_b - \tau_s - \frac{4}{3}(s_b - c_b) - \frac{2}{3}c_s \tag{A.2.10}$$

The point of intersection satisfy condition (A.2.4) if:

$$\alpha_b^P + \alpha_s^P < 2\sqrt{\tau_b\tau_s}$$
$$\iff 2\tau_s - \tau_b + \frac{4}{3}c_s + \frac{2}{3}(s_b - c_b) + 2\tau_b - \tau_s - \frac{4}{3}(s_b - c_b) - \frac{2}{3}c_s < 2\sqrt{\tau_b\tau_s}$$
$$\iff \tau_s + \tau_s - \frac{2}{3}s_b + \frac{2}{3}(c_b + c_s) < 2\sqrt{\tau_b\tau_s}$$
$$\iff s_b > \frac{3}{2}(\tau_b - 2\sqrt{\tau_b\tau_s} + \tau_s) + c_b + c_s$$
$$= \frac{3}{2}(\sqrt{\tau_b} - \sqrt{\tau_s})^2 + c_b + c_s \tag{A.2.11}$$

Now, condition (A.2.11) ensures that there exist parameter values that satisfy the three conditions for $\alpha_b, \alpha_s \in \mathbb{R}$. For there to exist $\alpha_b, \alpha_s > 0$ that satisfy the three conditions given that $s_s = 0$ it must also be that (A.2.4) and (A.2.6) are both satisfied for any $\alpha_b, \alpha_s > 0$. This is the case if (A.2.6) intersects the horizontal axis for a lower $\alpha_b$ than (A.2.4) when the conditions are represented in $(\alpha_b, \alpha_s)$-plane for given values of the other parameters. (A.2.4) intersects the horizontal axis for $\alpha_b = 2\sqrt{\tau_b\tau_s}$. Then, for there to exist $\alpha_b, \alpha_s > 0$ that satisfy the three conditions it has to be that:

$$\alpha_s^{A.2.6}(2\sqrt{\tau_b\tau_s}) = 3\tau_s + 2c_s - 4\sqrt{\tau_b\tau_s} < 0 = \alpha_s^{A.2.4}(2\sqrt{\tau_b\tau_s}) \tag{94}$$

Hence, given that $s_s = 0$, it is necessary that the following two conditions hold for there to exist solutions to the system of inequalities:

$$s_b > \frac{3}{2}(\sqrt{\tau_b} - \sqrt{\tau_s})^2 + c_b + c_s \tag{95}$$
$$3\tau_s + 2c_s - 4\sqrt{\tau_b\tau_s} < 0 \tag{96}$$

By symmetry, proposition 2 follows.

# B

## B.1 Profit maximization of a single-platform monopoly

Find the prices that solve the following problem:

$$\max_{p_s,p_b} \Pi_1^m = \frac{1}{\tau_b\tau_s - \alpha_b\alpha_s}\Big[(p_s - c_s)(\tau_b(s_s - p_s) + \alpha_s(s_b - p_b)) + (p_b - c_b)(\tau_s(s_b - p_b) + \alpha_b(s_s - p_s))\Big] - f_1$$

The first order conditions are:

$$\frac{\partial \Pi_1^m}{\partial p_s} = \frac{1}{\tau_b\tau_s - \alpha_b\alpha_s} \cdot [-2\tau_b p_s + \tau_b(s_s + c_s) - p_b(\alpha_b + \alpha_s) + \alpha_s s_b + \alpha_b c_b] = 0 \quad \text{(B.1.1)}$$

$$\frac{\partial \Pi_1^m}{\partial p_b} = \frac{1}{\tau_b\tau_s - \alpha_b\alpha_s} \cdot [-2\tau_s p_b + \alpha_s(c_s - p_s) + \alpha_b(s_s - p_s) + \tau_s(s_b + c_b)] = 0 \quad \text{(B.1.2)}$$

The second order conditions are:

$$\frac{\partial^2 \Pi_1^m}{\partial p_s^2} = -\frac{2\tau_b}{\tau_b\tau_s - \alpha_b\alpha_s} < 0 \quad \text{(B.1.3)}$$

$$\frac{\partial^2 \Pi_1^m}{\partial p_b^2} = -\frac{2\tau_s}{\tau_b\tau_s - \alpha_b\alpha_s} < 0 \quad \text{(B.1.4)}$$

$$|\mathbf{H}(p_b, p_s)| = \frac{\partial^2 \Pi_1^m}{\partial p_s^2} \cdot \frac{\partial^2 \Pi_1^m}{\partial p_b^2} - [\frac{\partial^2 \Pi_1^m}{\partial p_s \partial p_b}]^2 = \frac{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}{(\tau_b\tau_s - \alpha_b\alpha_s)^2} > 0 \quad \text{(B.1.5)}$$

By condition (EC) it follows that (B.1.3) through (B.1.5) hold and that the solution to the first order conditions are the profit maximizing prices.

Solving for $p_s$ and $p_b$ in (B.1.1) and (B.1.2):

$$p_s = \frac{1}{2\tau_b} \cdot [\tau_b(c_s + s_s) + \alpha_b c_b + \alpha_s s_b - (\alpha_b + \alpha_s)p_b] \quad \text{(B.1.6)}$$

$$p_b = \frac{1}{2\tau_s} \cdot [\tau_s(c_b + s_b) + \alpha_s c_s + \alpha_b s_s - (\alpha_b + \alpha_s)p_s] \quad \text{(B.1.7)}$$

Solving the equation system gives equilibrium monopoly prices:

$$p_s = \frac{(2\tau_b\tau_s - \alpha_b\alpha_s)(c_s + s_s) + \tau_s(\alpha_b - \alpha_s)(c_b - s_b) - \alpha_s^2 c_s - \alpha_b^2 s_s}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2} \tag{B.1.8}$$

$$p_b = \frac{(2\tau_b\tau_s - \alpha_b\alpha_s)(c_b + s_b) + \tau_b(\alpha_s - \alpha_b)(c_s - s_s) - \alpha_b^2 c_b - \alpha_s^2 s_b}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2} \tag{B.1.9}$$

Rewrite (B.1.8) as follows:

$$p_s = \frac{(2\tau_b\tau_s - \alpha_b\alpha_s - \alpha_s^2)c_s + (2\tau_b\tau_s - \alpha_b\alpha_s - \alpha_b^2)s_s + \tau_s(\alpha_b - \alpha_s)(c_b - s_b)}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}$$

$$= \frac{2\tau_b\tau_s - \alpha_b\alpha_s - \alpha_b^2}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2} s_s + \frac{(2\tau_b\tau_s - \alpha_b\alpha_s - \alpha_s^2)c_s + \tau_s(\alpha_b - \alpha_s)(c_b - s_b)}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}$$

$$= s_s - \frac{2\tau_b\tau_s - \alpha_b\alpha_s - \alpha_s^2}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2} s_s + \frac{(2\tau_b\tau_s - \alpha_b\alpha_s - \alpha_s^2)c_s + \tau_s(\alpha_b - \alpha_s)(c_b - s_b)}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}$$

$$= s_s + \frac{(c_s - s_s)(2\tau_b\tau_s - \alpha_b\alpha_s - \alpha_s^2) + \tau_s(\alpha_b - \alpha_s)(c_b - s_b)}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2} \tag{B.1.10}$$

Similarly, (B.1.9) can be rewritten as:

$$p_s = s_b + \frac{(c_b - s_b)(2\tau_b\tau_s - \alpha_b\alpha_s - \alpha_b^2) + \tau_b(\alpha_s - \alpha_b)(c_s - s_s)}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2} \tag{B.1.11}$$

From (B.1.10) and (B.1.11) we get that:

$$s_s - p_s = -\frac{(c_s - s_s)(2\tau_b\tau_s - \alpha_b\alpha_s - \alpha_s^2) + \tau_s(\alpha_b - \alpha_s)(c_b - s_b)}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2} \tag{B.1.12}$$

$$s_b - p_b = -\frac{(c_b - s_b)(2\tau_b\tau_s - \alpha_b\alpha_s - \alpha_b^2) + \tau_b(\alpha_s - \alpha_b)(c_s - s_s)}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2} \tag{B.1.13}$$

By using (B.1.12) and (B.1.13) in (56) and (57) we obtain the equilibrium network sizes:

$$n_s = \frac{2\tau_b(s_s - c_s) + (\alpha_b + \alpha_s)(s_b - c_b)}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2} \tag{B.1.14}$$

$$n_b = \frac{2\tau_s(s_b - c_b) + (\alpha_b + \alpha_s)(s_s - c_s)}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2} \tag{B.1.15}$$

It is optimal for the monopolist to serve the whole market when the parameter values are such that the respective equilibrium network sizes of buyers and sellers are at least 1 in equilibrium. This is the case when the following hold:

$$2\tau_b(s_s - c_s) + (\alpha_b + \alpha_s)(s_b - c_b) \geq 4\tau_b\tau_s - (\alpha_b + \alpha_s)^2 \tag{B.1.16}$$

$$2\tau_s(s_b - c_b) + (\alpha_b + \alpha_s)(s_s - c_s) \geq 4\tau_b\tau_s - (\alpha_b + \alpha_s)^2 \tag{B.1.17}$$

It is optimal for the monopolist to opt for a covered market when the total benefits from joining a platform are sufficiently high for both buyers and sellers relative to their degree of platform differentiation.

Now, since the market is defined by $n_b, n_s \in [0, 1]$, we derive the monopoly prices when (B.1.16) and (B.1.17) hold with equality:

$$2\tau_b(s_s - c_s) + (\alpha_b + \alpha_s)(s_b - c_b) = 4\tau_b\tau_s - (\alpha_b + \alpha_s)^2 \tag{B.1.18}$$

$$2\tau_s(s_b - c_b) + (\alpha_b + \alpha_s)(s_s - c_s) = 4\tau_b\tau_s - (\alpha_b + \alpha_s)^2 \tag{B.1.19}$$

Solving for $(s_s - c_s)$ in (B.1.19) we get that

$$s_s - c_s = \frac{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2 - 2\tau_s(s_b - c_b)}{\alpha_b + \alpha_s} \tag{B.1.20}$$

Using (B.1.20) in (B.1.18) yields:

$$\begin{aligned}
4\tau_b\tau_s - (\alpha_b + \alpha_s)^2 &= \frac{8\tau_b^2\tau_s - 2\tau_b(\alpha_b + \alpha_s)^2 - 4\tau_b\tau_s(s_b - c_b)}{\alpha_b + \alpha_s} + (\alpha_b + \alpha_s)(s_b - c_b) \\
\iff 4\tau_b\tau_s(\alpha_b + \alpha_s) - (\alpha_b + \alpha_s)^3 &= 8\tau_b^2\tau_s - 2\tau_b(\alpha_b + \alpha_s)^2 - 4\tau_b\tau_s(s_b - c_b) + (\alpha_b + \alpha_s)^2(s_b - c_b) \\
\iff (4\tau_b\tau_s - (\alpha_b + \alpha_s)^2)(s_b - c_b) &= (\alpha_b + \alpha_s)^3 - 2\tau_b(\alpha_b + \alpha_s)^2 - 4\tau_b\tau_s(\alpha_b + \alpha_s) + 8\tau_b^2\tau_s \\
&= ((\alpha_b + \alpha_s) - 2\tau_b)((\alpha_b + \alpha_s)^2 - 4\tau_b\tau_s) \\
&\iff s_b - c_b = 2\tau_b - \alpha_b - \alpha_s \\
\implies s_s - c_s &= \frac{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2 - 2\tau_s(2\tau_b - \alpha_b - \alpha_s)}{\alpha_b + \alpha_s} = 2\tau_s - \alpha_b - \alpha_s
\end{aligned}$$

Solving for $c_b$ and $c_s$:

$$c_b = s_b - 2\tau_b + \alpha_b + \alpha_s \tag{B.1.21}$$

$$c_s = s_s - 2\tau_s + \alpha_b + \alpha_s \tag{B.1.22}$$

Now using (B.1.21) and (B.1.22) in the equilibrium monopoly prices given in (B.1.8) and (B.1.9):

$$\begin{aligned}
p_s = & \frac{1}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}[(2\tau_b\tau_s - \alpha_b\alpha_s)(2s_s - 2\tau_s + \alpha_b + \alpha_s) \\
& + \tau_s(\alpha_b - \alpha_s)(-2\tau_b + \alpha_b + \alpha_s) - \alpha_s^2(s_s - 2\tau_s + \alpha_b + \alpha_s) - \alpha_b^2 s_s] \\
= & \frac{1}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}[4\tau_b\tau_s s_s - 2\alpha_b\alpha_s s_s - 4\tau_b\tau_s^2 + 2\alpha_b\alpha_s\tau_s + 2\tau_b\tau_s\alpha_b - \alpha_b^2\alpha_s + 2\tau_b\tau_s\alpha_s - \alpha_b\alpha_s^2 \\
& - 2\tau_b\tau_s\alpha_b + 2\tau_b\tau_s\alpha_s + \tau_s\alpha_b^2 - \tau_s\alpha_b\alpha_s + \tau_s\alpha_b\alpha_s - \tau_s\alpha_s^2 - \alpha_s^2 s_s + 2\tau_s\alpha_s^2 - \alpha_s^2\alpha_b - \alpha_s^3 - \alpha_b^2 s_s] \\
= & \frac{1}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}[(4\tau_b\tau_s - 2\alpha_b\alpha_s - \alpha_b^2 - \alpha_s^2)(s_s - \tau_s + \alpha_s)] \\
= & \frac{1}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}[(4\tau_b\tau_s - (\alpha_b + \alpha_s)^2)(s_s - \tau_s + \alpha_s)] \\
= & s_s - \tau_s + \alpha_s
\end{aligned}$$

$$\begin{aligned}
p_b = & \frac{1}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}[(2\tau_b\tau_s - \alpha_b\alpha_s)(2s_b - 2\tau_b + \alpha_b + \alpha_s) \\
& + \tau_b(\alpha_s - \alpha_b)(-2\tau_s + \alpha_b + \alpha_s) - \alpha_b^2(s_b - 2\tau_b + \alpha_b + \alpha_s) - \alpha_s^2 s_b] \\
= & \frac{1}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}[4\tau_b\tau_s s_b - 2\alpha_b\alpha_s s_b - 4\tau_s\tau_b^2 + 2\alpha_b\alpha_s\tau_b + 2\tau_b\tau_s\alpha_s - \alpha_s^2\alpha_b + 2\tau_b\tau_s\alpha_b - \alpha_s\alpha_b^2 \\
& - 2\tau_b\tau_s\alpha_s + 2\tau_b\tau_s\alpha_b + \tau_b\alpha_s^2 - \tau_b\alpha_b\alpha_s + \tau_b\alpha_b\alpha_s - \tau_b\alpha_b^2 - \alpha_b^2 s_b + 2\tau_b\alpha_b^2 - \alpha_b^2\alpha_s - \alpha_b^3 - \alpha_s^2 s_b] \\
= & \frac{1}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}[(4\tau_b\tau_s - 2\alpha_b\alpha_s - \alpha_b^2 - \alpha_s^2)(s_b - \tau_b + \alpha_b)] \\
= & \frac{1}{4\tau_b\tau_s - (\alpha_b + \alpha_s)^2}[(4\tau_b\tau_s - (\alpha_b + \alpha_s)^2)(s_b - \tau_b + \alpha_b)] \\
= & s_b - \tau_b + \alpha_b
\end{aligned}$$

In the case that it is optimal for the one-platform monopolist to serve the whole market, the monopoly prices will be given by:

$$p_s = s_s - \tau_s + \alpha_s \tag{B.1.23}$$

$$p_b = s_b - \tau_b + \alpha_b \tag{B.1.24}$$

## B.2 A perfectly symmetrical case

The system of inequalities in the perfectly symmetric case: $s_b = s_s = s$, $c_b = c_s = c$, $\alpha_b = \alpha_s = \alpha$, $\tau_b = \tau_s = \tau$. The system of inequalities reduces to:

$$\tau > \alpha \tag{B.2.1}$$

$$3\alpha - 3\tau - 2c + 2s \geq 0 \tag{B.2.2}$$

$$2s - 2c + 3\alpha - \frac{7}{2}\tau < 0 \tag{B.2.3}$$

Solving for $s$ in (B.2.2) and (B.2.3):

$$s < \frac{7}{4}\tau + c - \frac{3}{2}\alpha \tag{B.2.4}$$

$$s \geq \frac{3}{2}\tau + c - \frac{3}{2}\alpha \tag{B.2.5}$$

It follows that there are infinitely many solutions to the system of inequalities for parameter values that satisfy:

$$\alpha > 0 \tag{B.2.6}$$

$$\tau > \alpha \tag{B.2.7}$$

$$0 \leq c \leq s + \frac{3}{2}\alpha - \frac{3}{2}\tau \tag{B.2.8}$$

$$\frac{3}{2}\tau + c - \frac{3}{2}\alpha \leq s < \frac{7}{4}\tau + c - \frac{3}{2}\alpha \tag{B.2.9}$$